

On the multi-fractal structure of traded volume in financial markets

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In this article we explore the multi-fractal properties of 1 minute traded volume of the equities which compose the Dow Jones 30. We also evaluate the weights of linear and non-linear dependences in the multi-fractal structure of the observable. Our results show that the multi-fractal nature of traded volume comes essentially from the non-Gaussian form of the probability density functions and from non-linear dependences.

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I. INTRODUCTION

The intricate character of financial markets has been one of the main motives for the physicists interest in the study of their statistical and dynamical properties [1, 2]. Besides the asymptotic power-law behaviour for probability density function for price fluctuations, *the return*, and the long-lasting correlations in the absolute return, another important statistical feature observed is the return multi-fractal nature [3]. This propriety has been important in the establishment of analogies between price fluctuations and fluid turbulence [4] and the development of multiplicative cascade models for return dynamics too [5].

Changes in the price of a certain equity are basically related transactions of that equity. So, the *traded volume*, which is defined as the number of stocks that change hands during some period of time, is an important observable in the dynamics of financial markets. This observation is confirmed by an old proverb at Old Street that "It takes volume to make prices move" [6].

In previous works several proprieties of traded volume, V , either statistical or dynamical have been studied [7, 8, 9]. In this article, we present a study of the multi-fractal structure of 1-minute traded volume time series of the 30 equities which are used to compose the Dow Jones Industrial Average, DJ30. Our series run from the 1st of July until the 31st December of 2004 with a length of around 50k elements each. The analysis is done using the Multi-Fractal Detrended Fluctuation Analysis, MF-DFA¹ [10]. Besides the multi-fractal analysis we weight the influence of correlation, asymptotic power-law distribution and non-linearities in the multi-fractality of traded volume. Since we are dealing with intra-day series we have to be cautious with the well-known daily pattern which is often considered as a lacklustre propriety [12]. To that, we have removed that intra-day pattern of the original time series and normalised each element of the series by its mean value defining the normalised traded volume, $v(t) = \frac{V'(t)}{\langle V'(t) \rangle}$, where $V'(t) = \frac{V(t)}{\Xi(t')}$,

$\Xi(t') = \frac{\sum_{i=1}^N v(t'_i)}{N}$ and $\langle \dots \rangle$ is defined as the average over time (t' represents the intra-day time and i the day).

II. MULTI-FRACTALITY AND ITS COMPONENTS

A common signature of complexity in a system is the existence of (asymptotic) scale invariance in several typical quantities. This scale invariance, self-affinity for time series, can be associated to a single type of structure, characterised by a single exponent, H (the Hurst exponent) or by a composition of several sub-sets, each one with a certain local exponent, α , and all supported onto a main structure. The former is defined as a *mono-fractal* and the latter as

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¹ To evaluate the local trends we have used 5th-order polynomials. From this order ahead, we have a nearly polynomial-order-independent multi-fractal spectrum, contrarily to what happens with fitting polynomials of smaller order.

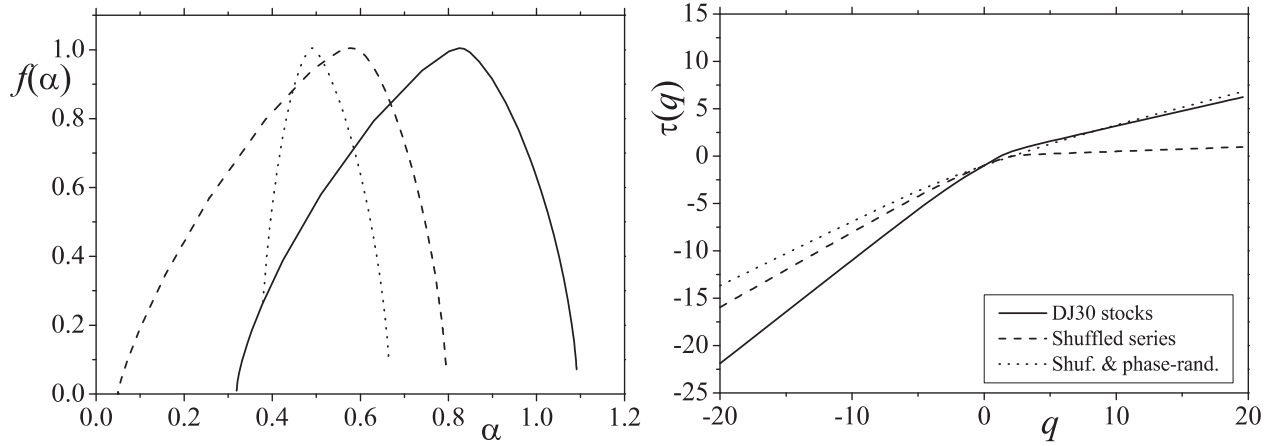


FIG. 1: Left panel: Multi-fractal spectra $f(\alpha)$ vs. α . Right panel: Scaling exponents $\tau(q)$ vs. q averaged over the 30 companies. The legend on the right is also valid for the left. The "original" and shuffled time series present a strong multi-fractal character, while the shuffled plus phase randomised time series presents a narrow width in α related with the almost linear dependence of $\tau_{sh-ran}(q)$ and also with the strong contribution of non-Gaussian PDF of traded volume to multi-fractality.

a *multi-fractal*. In this case the statistical proprieties of the various sub-sets are characterised by the local exponents α are related with a fractal dimension $f(\alpha)$ [11] composing the *multi-fractal spectrum*. To evaluate, numerically, this function we have applied the MF-DFA method [10]. For this procedure it was proved that the q -th order fluctuation function, $F_q(s)$, presents scale behaviour $F_q(s) \sim s^{h(q)}$. The correspondence between MF-DFA and the standard formalism of multifractals is obtained by,

$$\tau(q) = q h(q) - 1, \quad (1)$$

where $\tau(q)$ is the exponent of the generalised partition function. From Legendre transform, $f(\alpha) = q\alpha - \tau(q)$, we can relate $\tau(q)$ with the Hölder exponent [11], α . Thus, using the previous equation we get

$$\alpha = h(q) + q \frac{dh(q)}{dq}, \quad f(\alpha) = q[\alpha - h(q)] + 1. \quad (2)$$

In fig. 1(left) we display the $f(\alpha)$ spectrum (full line) obtained from averages for each q over the values of the 30 companies. In our analysis q runs from -20 to 19.5 . We have verified that $f(\alpha)$ presents a wide range of exponents from $\alpha_{\min} = 0.32 \pm 0.04$ up to $\alpha_{\max} = 1.09 \pm 0.04$, corresponding to a deep multi-fractal behaviour. For $q = 2$ we have obtained $h(2) \equiv H = 0.71 \pm 0.03$ which agrees with strong persistence previously observed [7]. From our time series we can define new and related ones that can help us to quantify which factors contribute the most to the multi-fractal character of this observable. Among these factors we name: linear correlations, non-linear correlations and power-law-like PDF. To that, we have shuffled the elements (within each time series) and from these series we have computed $h_{shuf}(q)$. From these uncorrelated time series we have created another set by randomising the phase in the Fourier space. Afterwards, we have applied the inverse Fourier transform to come back to the time variable. These new series have Gaussian stationary distribution and scaling exponent $h_{sh-ran}(q)$.

In fig. 1(left) we see that these two series also present multi-fractal spectrum, although the shuffle series has a wider spectrum than the shuffled plus phase randomised series. Concerning the Hurst exponent, $h(2) = H$, we have obtained $H = 0.49 \pm 0.03$ for shuffled and $H = 0.5 \pm 0.03$ for shuffled plus phase randomised series. Considering error margins, these values are compatible with $H = 1/2$ of Brownian motion. Furthermore, we have made a set of only phase randomised time series for which we have obtained $H = 0.7 \pm 0.03$. From this values we have concluded that correlations have a key role in the persistent character of traded volume time series.

Using $F_q(s)$ scaling relation for the three series [10] and assuming that all the factors are independent, we can quantify the influence of correlations, $h_{cor}(q) = h(q) - h_{shuf}(q)$, the influence of a non-Gaussian PDF, $h_{PDF}(q) = h_{shuf}(q) - h_{sh-ran}(q)$, and the weight of non-linearities, $h_{sh-ran}(q) \equiv h_{nlin}(q) = h(q) - h_{cor}(q) - h_{PDF}(q)$. The multi-fractality of a time series can be analysed by means of the difference of values between $h(q_{\max})$ and $h(q_{\min})$, hence

$$\Delta h = h(q_{\min}) - h(q_{\max}) \quad (3)$$

it is a suitable way to characterise multi-fractality. For a mono-fractal we have $\Delta h = 0$, *i.e.*, linear dependence of $\tau(q)$ with q . In fig. 1(right) we have depicted $\tau(q)$ for several time series from which we have computed the various Δh . The results obtained are the following: $\Delta h = 0.675$, $\Delta h_{cor} = 0.027$, $\Delta h_{PDF} = 0.445$, and $\Delta h_{mlin} = 0.203$. As it can be easily concluded the influence of linear correlations in traded volume multi-fractal nature is minimal with Δh_{cor} corresponding to 4% of Δh . This value is substantially smaller than the influence of Δh_{mlin} which corresponds to 30% of Δh . Our result is in perfect accordance with another previous result of us [9] where, using a non-extensive generalised mutual information measure [13], we were able to show that non-linear dependences are not only stronger but also more resilient than linear dependences (correlations) in traded volume time series. Last but not least, from the values of Δh we have verified that the main factor for the multi-fractality of traded volume time series is its non-Gaussian, generalised q -Gamma [7, 8, 9], probability density function with a weight of 66% in Δh . Moreover, we have verified that the behaviour of $\tau(q)$ for $q > 0$ is quite different from the $q < 0$, which is also visible in the strong asymmetry of $f(\alpha)$. This could indicate that large and small fluctuations appear due to different dynamical mechanisms. Such scenario is consistent with the super-statistical [14] approach recently presented [8, 9] and closely related with the current non-extensive framework based on *Tsallis entropy* [15]. Within this context and bearing in mind the relation $1/(1 - q_{sens}) = 1/\alpha_{min} - 1/\alpha_{max}$ [16], we conjecture that, for traded volume, the sensitivity to initial conditions may be described by $\xi = [1 + (1 - q_{sens}) \lambda_{q_{sens}} t]^{\frac{1}{1 - q_{sens}}}$ with $q_{sens} = 0.55 \pm 0.08$.

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