Hedging Currency Risks – Dynamic Hedging Strategies Based on O & A Trading Models

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1 Introduction

Hedging problems arise whenever an investor, for example a fund manager or a commercial organization, is holding foreign assets such as foreign securities over a period of time. The foreign assets are denominated in a foreign currency. The investor measures the performance of his investment in terms of his home currency. The foreign assets have a degree of volatility in terms of their own currency. Due to the foreign exchange rate movements (FX) the volatility is, however, higher when expressed in terms of the investor's home currency. This implies additional risk.

The currency risk can be hedged by taking a short position in the currency of the security. If this short position is constant over time, then this is *static* hedging. If it is changed over time, the hedging strategy is *dynamic*.

In this document, dynamic hedging of the foreign assets is proposed using an O & A trading model portfolio. The O & A trading models are the result of scientific research that has been published in economic journals: (Müller et al., 1990), (Dacorogna et al., 1993). A description of the O & A trading models has been published in (Pictet et al., 1992). In the documents (Dacorogna, 1993b) and (Dacorogna, 1993a), the performance of these models is presented and discussed. Many banks are using the O & A real-time information system and are able to execute the O & A trading model recommendations on behalf of any investor.

The optimal hedging strategy depends upon certain decisions of the investor: choice of the foreign countries of the investment, frequency of investment changes, limits of risk and exposure in the dynamic hedging strategy, ... The document includes a questionnaire for the investor. O & A has the means of designing a suitable hedging strategy based on the investor's basic decisions.

2 The basic foreign investment

Before solving the hedging problem, we have to specify the home currency and the nature and size of the foreign investment to be hedged.

The investor has assets in n foreign countries, corresponding to n currencies. We denote the current values of all these foreign assets by a_j, where j is the index of the currency, and the measuring scale for all assets is the home currency (as usual in the book-keeping of investments). The index j of the currency is between 1 and n; the "zeroth" currency with $j = 0$ is the home currency of the fund.

The hedging strategy and its formulas are illustrated by one specific example in this document. This example is a Swiss investor with the Swiss Franc (CHF) as his home currency and a foreign investment with a total value of 100'000'000 CHF. The foreign countries for the investment are the USA, Japan, Germany, France, and Britain ($n = 5$). The investment in these countries may be in equity. Each of the five countries has an investment share of 20%, that is an amount of $a_j = 20'000'000$ CHF for each. We assume that this foreign investment is stable over a certain period. The hedging strategy presented in the following sections can equally well be applied to other types of investments and other home currencies.

3 Static hedging

For an investor who owns a foreign asset such as equity of value a_j, hedging means holding a short position of size $-s_j$ in the underlying jth foreign currency. He will try to minimize the volatility of the value of his total position which is subject to FX rate fluctuations. The hedging ratio h_j is defined as

$$h_j = \frac{s_j}{a_j} \tag{3.1}$$

A study (Froot, 1993) has shown that choosing the best hedging ratio h, the one to minimize the total volatility, is not trivial and depends on the time horizon of the investor. For short-term investors, the best h is 1 or slightly less, for long-term investors who hold their position over many years, the best choice of h is about 0.35. $h = 1$ is called full hedging, $h = 0$ is no hedging.

Static hedging means that the hedging ratio h stays constant over time. Before discussing dynamic hedging strategies, the investor has to take a basic decision on the best static hedging ratio h_{stat}. This decision depends on several factors. Our typical recommendation, which is made in view of the results of (Froot, 1993), is to choose $h_{\text{stat}} = 0.5$, that is a 50 percent hedge. In the example of section 2, this means permanent, short positions of a size of 10'000'000 CHF in all the five foreign currencies (USD, JPY, DEM, FRF, and GBP). To be precise: a short position in, say, FRF means to be short in French Francs against the home currency, the Swiss Franc (in the cross rate to the home currency rather than the US Dollar rate).

As an alternative to the static hedging with $h_{\text{stat}} = 0.5$, we introduce a dynamic hedging strategy using the O & A trading models.

4 Dynamic hedging

The static hedging strategy determines the equilibrium point or neutral point of the dynamic hedging strategy. If all the O & A trading models used for dynamic hedging are neutral, the hedging is determined by the static equilibrium point, or in the above example by $h_{\text{stat}} = 0.5$.

The O & A trading model technology is described in (Pictet et al., 1992). If an O & A trading model used in the hedging strategy takes a long or short position, the overall hedge changes. If this model is followed with, say, 10'000'000 CHF, and goes "Short-1", the overall position in one currency changes. In the example of section 2 it is –10'000'000 – 10'000'000 = –20'000'000 CHF (a short position corresponds to a full hedge) and the hedging ratio becomes $h = 1$. If this model goes "Long+1.0", the overall position becomes 10'000'000 – 10'000'000 = 0 CHF (no hedging) and the hedging ratio becomes $h = 0$. In this example, the changes are between a complete hedge and no hedge. If the trading model is followed with a higher dealing limit than 10'000'000 CHF, the fluctuation of the hedging ratio becomes even larger.

The recommended dynamic hedging strategy is based on more than one O & A trading model: a portfolio of trading models with different FX rates, including cross rates. Portfolios of trading models have a better behavior over time than individual models: the risk of large drawdowns (loss phases) is much lower. The overall exposure and the hedging ratio h can thus depend on more than one trading model. This problem is dealt with in the next sections.

5 Dynamic hedging constraints

The O & A trading models are known to be profitable in the long run. An investor might thus choose to follow these models with a very high leverage in order to generate a high profit. In this case, the fluctuations of the hedging ratio would also be very high. A strategy that exceeds certain limits of leverage, respectively the hedging ratio, can no longer be termed a hedging strategy (where hedging stands for limiting the risk), it is rather active FX speculation. An aggressive trading strategy may be interesting for some investors, but it is outside the scope of this document which is restricted to real hedging strategies.

For a similar reason, we want to exclude all FX rates with currencies that are neither the home currency nor belong to the primary foreign investment. Thus, we avoid introducing new currency risks other than those already implied by the primary foreign investment.

The number of trading models in the portfolio is not fixed, and the home currency may or may not be one of the currencies of the trading models. There may be more than one trading model with the same currency or even the same currency pair. (A currency corresponding to a certain foreign investment may be missing in the trading model portfolio, but then, the risk related to this currency is hedged only statically, not dynamically).

The hedging portfolio is chosen to have m O & A trading models, where the choice of m does not directly depend on the number n of the currencies. Each trading model is used for dealing with a certain amount of money c_i, a certain credit limit, which corresponds to "gearing 1" of the trading model. i is the index of the trading model. The ith trading model has two currencies, the "home" currency $C_{\mathrm{h}\,i}$ and the "foreign" currency $C_{\mathrm{f}\,i}$. The terms "home" and "foreign" are meant here just for the FX rate of the trading model, they do not refer to the basic investment. For example: in a USD-DEM trading model, USD is the "foreign" and DEM the "home" currency. The two currencies must be chosen among the available currencies:

$$0 \;\leq\; C_{\mathrm{h}\,i} \;\leq\; n \;\;,\;\; 0 \;\leq\; C_{\mathrm{f}\,i} \;\leq\; n \;\;,\;\; C_{\mathrm{h}\,i} \;\neq\; C_{\mathrm{f}\,i} \tag{5.1}$$

The last equation simply states that the two currencies of the trading model must be different.

Now, the total possible exposure e_j in the jth foreign currency can be computed:

$$e_j \;=\; \sum_{i=1}^{m} \gamma_{ij} c_i \;\;,\;\; \text{for } j \;=\; 1 \ldots m \tag{5.2}$$

where γ_{ij} is a factor that determines whether the jth currency is used in the ith trading model:

$$\gamma_{ij} \;=\; \begin{cases} 1 & \text{if } C_{\mathrm{h}\,i} \;=\; j \;\text{ or }\; C_{\mathrm{f}\,i} \;=\; j \\ 0 & \text{otherwise} \end{cases} \tag{5.3}$$

A maximum possible exposure e_j means that the hedging ratio h_j can fluctuate between $h_{\mathrm{stat}} - e_j/a_j$ and $h_{\mathrm{stat}} + e_j/a_j$.

The maximum exposures e_j in all currencies are limited in a reasonable hedging strategy. Exposures limit can be defined globally or currency-wise:

FX rate O & A trading model number	DEM-JPY 40	DEM-JPY 50	USD-CHF 50	USD-FRF 40	GBP-USD 40	USD-DEM 50
amount of money per model (in CHF)	5'000'000	5'000'000	5'000'000	5'000'000	2'500'000	2'500'000

Table 1: Composition of a portfolio of O & A trading models suited to the dynamic hedging of the foreign assets of a Swiss investor as described in the example of section 2.

1. Global exposure limit: a limit for the total exposure in all foreign currencies together. The size of the total foreign investment, a, is probably the largest reasonable choice for the maximum range of dynamic hedging. The maximum amount of money allocated to the O & A trading model portfolio is half this size, $a/2$, because the O & A trading models have a total dealing range of two gearing steps, from Short-1 to Long+1. Some investors may prefer narrower global exposure limits.

2. Currency-wise exposure limits: the exposure in each foreign currency alone must stay within certain limits. The natural choice is to keep the hedging ratio h_j for each of the n currencies between 0 (no hedging) and 1 (full hedging), or $e_j \leq a_j/2$. In our example, we allow for a moderately higher currency-wise exposure limit: $e_j \leq 0.75 a_j$. This is possible because the risk is still limited by the global hedging limit.

6 The choice of the trading model portfolio for dynamic hedging

Within the given constraints, the set of allowed currencies and the exposure limits, O & A can help to identify an optimal trading model portfolio with a high yearly return, minimum risk (low maximum drawdown), and a balanced, well diversified mixture of different models.

For the example of section 2, the O & A model portfolio of table 1 can be selected. This portfolio is followed with a total amount of 25'000'000 CHF. It is based on the currently available O & A on-line trading models which are described in (Pictet et al., 1992), (Dacorogna, 1993b), and (Dacorogna, 1993a). In the future, O & A will make new trading models available. Therefore, O & A will recommend modified portfolios with some of these new trading models within the same hedging limits.

The sum of 25'000'000 CHF invested in the portfolio is clearly lower than allowed by the maximum global limit $a/2 = 50'000'000$ CHF. Yet, the exposure in one individual currency, the USD, can be quite high because most of the O & A trading models are based on FX rates against the USD: $e_j = 15'000'000$ CHF, following equation 5.2. If all the models with a USD rate happen to be long in the USD, then the exposure is +15'000'000 CHF from the neutral point. This is exactly the allowed currency-wise limit of $0.75 a_j$. For the USD, this limit can sometimes (but not very frequently) be reached. For all other foreign currencies, the exposure stays below the currency-wise limit.

7 Performance of the dynamic hedging portfolio

First, the performance of the portfolio alone is considered. In the next section, this performance will be treated in the framework of a complete hedging strategy, including static hedging.

O & A has the tools to directly analyze the historical performance of trading model portfolios. In the literature, performance and risk of trading models are sometimes analyzed by considering the correlation between different trading models. This is not necessary in our case as we rely on more direct portfolio performance measures such as the mean return, the Sharpe ratio, the maximum drawdown and others.

The portfolio of table 1, selected for the investment example of section 2, has the following properties, computed in a historical test over more than six years:

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Mean monthly return: 0.9%.  Statistics around this mean: std dev.: 1.4%,
monthly Sharpe ratio: 0.67, annualized Sharpe ratio: 2.32
Percentage of negative monthly returns: 25.0%, profit/loss ratio: 3.00
Statistics includes transaction costs but no interest gains.
Testing period, start: 01.12.86, end: 01.03.93
Out-of-sample period, start: 01.12.89, end: 01.03.93
Percentage of time in full exposure: 7.8%, annualized turn-over: 41.2
Drawdown, max: 2.7%, period: 57.4 days
Annualized return: 11.28%
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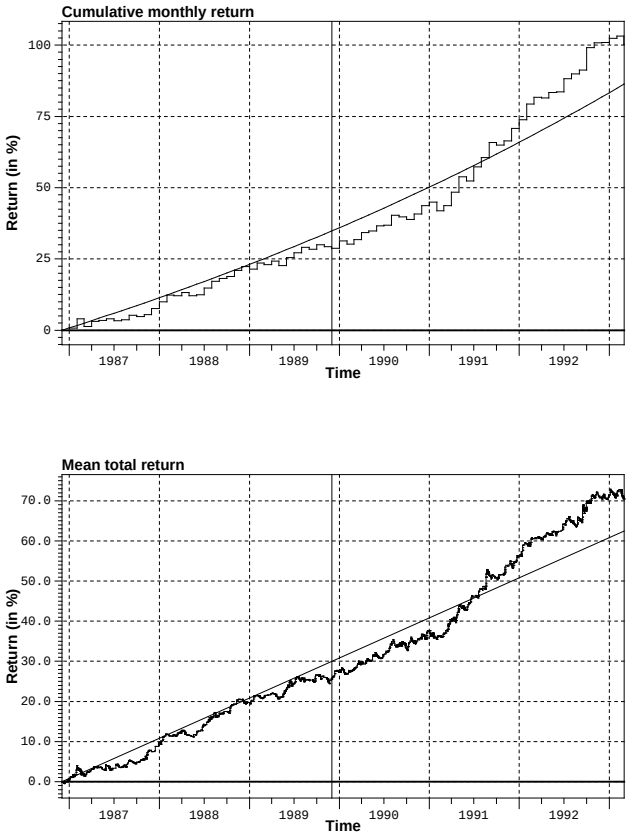
The yearly return of this portfolio, based on these historical results, is of 11.3%, with a standard deviation of 4.8%. This figures refer to the amount of capital of 25'000'000 CHF invested in the trading model portfolio. In absolute figures, the yearly return is thus 2'800'000 CHF with a standard deviation of plus or minus 1'200'000 CHF. In figure 1, the behavior of the portfolio over the years is also shown in graphic form.

Can the figures based on more than six years of historical analysis also be expected for the future? At O & A, a more cautious expected yearly return has been defined as the historical average yearly return minus one third of the standard deviation. This has been introduced because of the risk of unknown structural breaks in the market behavior. This expected yearly return has the value of 2'400'000 CHF with a standard deviation of plus or minus 1'200'000 CHF.

Note that these performance figures do not contain any interest on capital. It refers to a pure hedging portfolio. Interest rate differentials between currencies have also been neglected in the return computation, but this implies only a small error because of the relatively high dealing frequency of O & A trading models. The performance figures are realistic as they account for the transaction costs (bid-ask spreads). They refer to an unleveraged portfolio of unleveraged trading models.

8 Dynamic hedging performance compared to simple benchmarks

The performance of the dynamic hedging portfolio has to be compared to that of another strategy. It can be compared to static hedging with constant positions. There are three static hedging strategies that can be used for this comparison:



The O & A trading model portfolio return, cumulative (above) and total (below), in the test period from 1 Dec 86 to 1 Mar 93. The straight curves represent an annualized return of 10%. The vertical line indicates the limit of the in/out-of-sample test. Portfolio composition: DEM-JPY-40, DEM-JPY-50, GBP-USD-40 (0.5), USD-CHF-50, USD-DEM-50 (0.5), USD-FRF-40. Statistical results: see text.

Figure 1: Historical return of the dynamic hedging portfolio

1. **Full hedging**: constant short positions of size a_j in all five foreign currencies. In our example: 20'000'000 CHF.

2. **Half hedging**: constant short positions of size $a_j/2$ in all five foreign currencies. In our example: 10'000'000 CHF. This is the actually recommended static hedging, see section 3.

3. **No hedging**: no constant short positions in any foreign currency.

These three static hedging strategies are called the benchmark strategies; they have no dynamic component.

The performance of all three benchmark strategies has been analyzed over a historical test period of more than six years. This period is the same as for measuring the dynamic hedging portfolio performance. In static hedging strategies, we establish permanent short positions in foreign currencies. Therefore, the interest rate differentials between the currencies play a big role and are taken into account here.

Among the three benchmark strategies, the "no hedging" strategy was best in our historical test period. This does not mean that it will the best strategy in the future, it just reflects the fact that the Swiss Franc was a worse investment in this period than most other currencies.

Performance of the dynamic hedging strategy

benchmark for comparison	compared to full hedging	compared to half hedging	compared to no hedging
difference in the mean yearly return	4'200'000	2'800'000	1'400'000

Table 2: Performance of a dynamic hedging strategy with O & A trading models, for the example of a Swiss investor described in section 2 and table 1. The difference between the yearly return of dynamic hedging and the yearly return of three different static benchmark strategies is shown. The yearly return figures are averages over the historical test period 1 Dec 86 – 1 Mar 93 and expressed in Swiss Francs.

In table 2, the historical performance of dynamic hedging – on top of static half-size hedging as described in section 3 – is compared with the performance of the three benchmark strategies. All the performance figures are positive: dynamic hedging compares favorably with all static benchmark strategies. These performance figures refer to only the hedging strategy. The investor can of course expect also a return on his primary foreign investment, for example dividends of foreign stocks, but this primary return is outside the considerations of this hedging document.

The smallest performance advantage of our proposed dynamic hedging strategy is observed against the "no hedging" strategy, because this benchmark strategy was quite successful in the historical test period where the Swiss Franc was a worse investment than most foreign currencies. In the future, these things may be different.

More relevant than the historical performances of table 2 are the expectations for the future. O & A is estimating the future performance of its portfolio performance slightly lower than the historical one, because of the risk of unknown structural breaks in the market behavior. It can also be expected that the difference between the three benchmark strategies will no longer be so large as in the test period of table 2.

9 Varying the investment and the hedging strategy over time

Until now, we have assumed that the foreign investments to be hedged (the a_j values) are constant over time. The values of the foreign assets and their underlying currencies are not stable, though. An investor will find the values slowly drifting away from the initial settings of a_j, to the positive or the negative side. This will slowly push also the hedging strategy out of its equilibrium.

Moreover, the investor may also actively vary his allocation of foreign investments. He may take out some money from a country with currency j and put it as an additional investment into an asset of the country with currency j'. If he is doing so without modifying the hedging strategy, the jth currency will become overhedged and the currency with index j' underhedged. This is true for static hedging as well as the size of dynamic hedging.

These are reasons to revise the hedging strategy from time to time, once every year or more frequently. Another reason arises from the progress in trading model development. O & A will make their new trading models and portfolios available for hedging after these have been successfully tested in-house.

A strategy revision might be organized as follows:

1. The investor assesses the likely average investment scheme for the next period, e. g. the next year. This defines the a_j values for the hedging strategy.
2. The hedging advisor redesigns the best dynamic hedging strategy for these new a_j values.
3. The difference between the current static and dynamic hedging positions and the desired positions following the new strategy is measured. With a minimum set of transactions, the positions are corrected to the new desired positions.

10 Questionnaire: basic investment decisions

This questionnaire may be helpful in the specification and design process of a dynamic hedging strategy. The questions refer to investment decisions discussed in the previous sections.

- What is the basic foreign investment? In which countries? Which quantities?
- How often will this allocation scheme be changed? How strongly will it vary? Can large changes occur, for example that all the money is taken completely out of one country and invested in another one?
- How often is it possible to revise and re-adapt the hedging strategy?
- What should be the static hedging, in other words the neutral point of the dynamic hedging? Is the proposed solution acceptable? It means keeping a constant, short currency position of 50% the size of the investment in the corresponding country.
- What is the global exposure limit for the dynamic hedging with the portfolio of O & A trading models?
- What is the currency-wise exposure limit for dynamic hedging?
- Which is the relevant benchmark strategy for computing the performance of the dynamic hedge?

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