# On the intra-daily performance of GARCH processes

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**Abstract**

*In this paper, we show that the use of an alternative time-scale can eliminate the inefficiencies in the estimation of a GARCH model caused by intra-daily seasonal patterns. Even so, however, the temporal aggregation properties of the GARCH model do not hold at the intra-daily frequencies, revealing the presence of several time-horizons components. Besides, distinct characteristics were identified in the very short (less than 2 hours) and the very long (several months) run. Finally, the out-of-sample predictive power of GARCH for the volatility was found to be lower than the historical volatility itself implying the presence of other sources of heterogeneity.*

# 1 Introduction

The Auto Regressive Conditional Heteroskedastic (ARCH) model (Engle, 1982) and its Generalised version (GARCH) (Bollerslev, 1986) are now not only widely used in the FX literature (Bollerslev et al., 1992) but also as the basic framework for empirical studies of the market microstructure such as the impact of news (Goodhart and Figliuoli, 1991; Goodhart et al., 1993) and government interventions (Goodhart and Hesse, 1993; Peiers, 1994), or inter- and intra-market relationships (Engle et al., 1990; Baillie and Bollerslev, 1990). The main assumption behind this class of models is the relative homogeneity of the price discovery process among market participants at the origin of the volatility process. In other words, the conditional density of one GARCH process can adequately capture the information of the news. In particular, GARCH parameters for the weekly frequency theoretically derived from daily empirical estimates are usually within the confidence interval of weekly empirical estimates (Drost and Nijman, 1993).

However, several empirical evidences seem at odds with this homogeneous view of the market. First, the long memory of the volatility (Dacorogna et al., 1993; Ding et al., 1993) indicates the presence of several market components corresponding to several time-horizons. Note that this property of the volatility has already been successfully incorporated in the GARCH setting as the Fractionally Integrated GARCH (Baillie et al., 1993). Second, at the intra-daily frequency, round-the-clock time series reveal seasonal patterns that reflect, among others, the geographical dispersion of the traders, concentrated in three main geographical areas, Asia, Europe and America. Although the first investigations of the effect of these different geographical locations seemed to indicate that news would just spread out around the world (the so-called meteor shower hypothesis (Engle et al., 1990)), recent empirical studies (see Guillaume et al. (1994) for a survey) brought to light the presence of these market components. Third, exchange rates movement are not necessarily related to the arrival of news when looked at the intra-daily frequency (Goodhart, 1989), reflecting the fact that intra-day traders may have other constraints and objectives than, for example, longer term traders. Fourth, at the extremely high frequencies, FX rates exhibit distinct characteristics due to the price formation process (Guillaume et al., 1994).

In this paper, we investigate the importance of this heterogeneity for the modelling of the foreign exchange (FX) markets using the GARCH setting. More specifically, we show that estimates of a GARCH process with data in physical time are likely to be spurious, even though estimates for one particular frequency seem to be reasonable. Estimates are only consistent when

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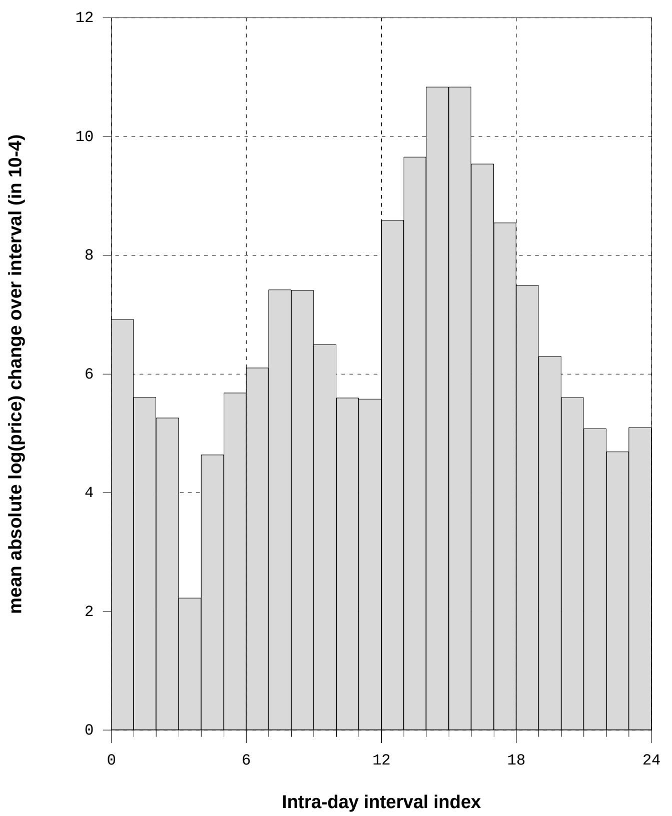
the seasonal patterns are taken into account. However, even when these seasonal patterns are accounted for, the aggregation properties of the GARCH model break down at the intra-daily frequencies, revealing the presence of traders with different thresholds or risk profiles. In addition to the presence of these different categories of traders, one also observes the effects of the price formation process at frequencies higher than the 1 1/2 hour time interval. At the other extreme, the instability of coefficient estimates over different sub-periods of 6 months suggests the presence of a seemingly random component at very long time-intervals. Finally, we show that these mis-specifications of the GARCH process result in its very poor out-of-sample predictive power for the volatility relative to the (unconditional) historical volatility.

The remainder of this paper is divided as follows. In section 2, we present the data and describe regular patterns such as the seasonal deterministic components and its fractal structure. A different time scale which integrates these patterns is discussed in section 3. In section 4, we present results of GARCH estimates for several frequencies in both physical time and deseasonalised time. The accuracy of the GARCH model is also tested for these different frequencies. Section 5 concludes the chapter.

## 2 Description of the data




Histogram of the hourly average of the absolute price change for the USD/DEM. The x-axis is the hour of the day in GMT. The sampling period runs from 1.01.1987 to 31.12.1993.

Figure 1: Intra-day seasonality

The foreign exchange (FX) market is a round-the-clock market with traders situated all over the world. Although the FX market does not officially close, traders are typically active during the business hours corresponding to one of the three major trading centers, namely, Europe, America and East-Asia. Trading usually occurs in two stages: the publication of price quotes through one of the main electronic data vendors such as Reuters or Telerate and the actual trade negotiated over the telephone. Since the actual transaction prices are generally not available, the FX rates time series consist of these electronic price quotes. Daily FX rates, for instance, are the average of the four or five last quotes by major banks around a particular hour of the day, for example 4.00 p.m., Greenwhich Mean Time (GMT). Daily FX rates are thus extracted from the intra-daily time series of price quotes.

In this paper, we use a 7-years data bank of such intra-daily quotes for the period from 1.01.1987 to 31.12.1993 for the major currencies against the USD, that is the DEM, JPY, GBP, FRF and CHF. As in Müller et al. (1990) and Dacorogna et al. (1993), linear interpolation over time is used to determine price values within data holes and to generate regularly spaced time series. A minimum time interval of 10 minutes was taken to avoid price uncertainty (Guillaume et al., 1994). Price are computed as the average of the logarithm of the bid and ask prices ($x(t_i) = [\log p_{bid}(t_i) + \log p_{ask}(t_i)] / 2$ where $t_i$ indicates a fixed sampling) and returns as the corresponding price changes ($r(t_i) = [x(t_i) - x(t_i - \Delta t)]$). In Table 1 in the Appendix, we give a summary of the properties of these time series for the main intervals as well as the weekly frequency. One very apparent feature of this table is the very fast increase of the kurtosis of the returns with increasing sampling frequency. This major characteristic of FX returns is the direct consequence of the non-convergence of the fourth moment of the distribution (see Dacorogna et al., 1994). This already suggests that the GARCH process might not provide an adequate description of the FX returns as it relies on the assumption of the existence of the fourth moment of the distribution.

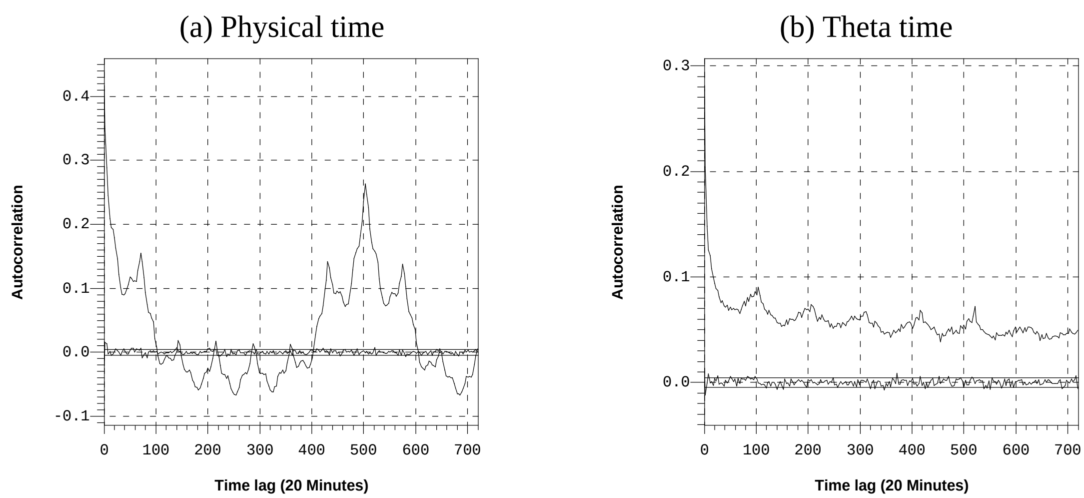



Figure 2: Autocorrelation of the absolute price changes in physical time (with seasonal peaks) and in $\vartheta$-time (with long memory effects) for the GBP/USD. The sampling period runs from 1.01.1987 to 31.12.1993.

The second major characteristic of intra-daily returns, the strong seasonal deterministic structure of the intra-daily volatility is illustrated in Figures 1 and 2(a). The presence (or absence) of the different geographical markets can easily be seen on Figure 1 with the decline in the activity around 4.00 GMT when most of the Japanese trader are out for lunch and the strongest intensity in the afternoon when both the American and European traders are present. In Figure 2(a), much stronger autocorrelation patterns can be seen at lags that are integer multiples of seasonal patterns. Although several studies of the micro-structure of the market have already tried to establish a causal link between these different geographical markets and the conditional heteroskedasticity of the returns (Guillaume et al., 1994), they did not make the distinction between conditional and seasonal heteroskedasticity in their empirical models. One reason for this short-coming of previous studies is that they concentrate their analyzis on data at one particular frequency. And as will be demonstrated in section 4, one can easily overlook this deterministic seasonal feature as empirical estimations for one particular frequency may appear to be reasonable.

A third characteristic of intra-daily returns is their fractal structure. Whereas the previous feature of FX returns mainly characterized the geographically heterogeneity of the traders, the fractal structure of the markets establishes a link between traders operating at very different time-horizons, ranging from market makers acting at very high frequency to long term investors trading over much longer horizons of several weeks or even months. This fractal structure is characterized by the following scaling law linking the average volatility over a time interval to the size of this interval. Formally, we have (Müller et al., 1990):

$$\overline{|r(t_i)|} = \left(\frac{\Delta t_i}{\Delta T}\right)^{1/\mathrm{E}} \tag{2.1}$$

In the above relationship the absolute value of the returns was taken as a measure of the volatility but the same relationship is valid for their squared value. Taking the logarithm on both sides, one can estimate the drift exponent – 1/E –. This drift exponent is a constant whose value is quite stable over the years and across the currencies (Guillaume et al., 1994). The parameter $\Delta T$ depends upon the FX rate and is obtained from the regression constant of the logarithmic form of eq. 2.1. This relationship provides thus a way to characterize the average (or expected) volatility for one particular frequency. This empirical law will be used in the next section to model the deterministic seasonal pattern of the volatility.

A fourth characteristic of intra-daily returns is the price formation process which takes place at the highest frequencies (Guillaume et al., 1994). In particular, the high short term negative autocorrelation makes the nonlinear structure of FX rates even more complex (Guillaume, 1994). Moreover, the size of the spread is comparable to the size of price changes to frequencies up to 80 minutes.

# 3 Alternative time scales

In empirical studies of daily FX rates, one usually assumes implicitly a different time scale than the physical time scale by omitting observations corresponding to the week-ends when the market is mostly inactive. This alternative time scale corresponds therefore to what one would call a business time scale. In a similar way, we already implicitly discussed two different time scales in the case of intra-daily data: regularly and irregularly – tick by tick – spaced prices. To avoid uncertainty on the price, we took a minimum time interval of 10 minutes to compute returns in physical time. Observations during week-end are, however, rather sparse, causing an abrupt change in the shape of the autocorrelation function of the volatility as is shown in Figure 2(a). A short-cut to treat this problem would be the definition of a business time scale in the same way as for daily observations by simply omitting observations corresponding to the week-ends when the markets are virtually closed, that is from Friday 22.30 GMT to Sunday 22.30 GMT. In the remainder of this paper, we will refer to this time scale without the week-end observations as the business time scale.

A more proper way to fully account for the variation in the presence (or absence) of the market agents is to directly model the deterministic seasonal patterns corresponding to the presence of the different geographical markets. Such a model was presented in Dacorogna et al. (1993) where the resulting time scale was called the Theta ($\vartheta$) time scale. To construct this $\vartheta$-time scale, the inverse of the scaling law is applied to the hourly average volatility for each hour of the week resulting in the following activity statistics:

$$a_{stat,i} \equiv \frac{\Delta T}{\Delta t} (\overline{|r(t_i)|})^{\mathrm{E}} \tag{3.1}$$

where $\Delta t = 1$ h and the index $i$ refers to the hour of the week ($i = 1, ..., 168$). An activity function $a(t)$ is then fitted to the results of the statistics $a_{stat,i}$. This activity function is divided into three components corresponding to the three main geographical FX markets – East-Asia, Europe and America –. Each of these markets is described by an activity variable $a_0$ corresponding to a constant base level during the closing hours and an activity variable $a_k(t)$ describing the activity during the opening hours of the corresponding market ($k = 1, 2, 3$). The $\vartheta$-time is then the time integral of the worldwide activity:

$$\vartheta(t) \equiv a_0 \ (t - t_0) + \sum_{k=1}^{3} \int_{t_0}^{t} a_k(t') \ \mathrm{d}t' \tag{3.2}$$

The activity variable is normalized in such a way that $\vartheta$-time can be measured in the same units as physical time (e. g. hours, days, weeks); one full week in $\vartheta$-time corresponds to one week in physical time. With the exception of the day of the week effect, this $\vartheta$-time scale thus explicitely takes into account most of the deterministic seasonal patterns; that is, the presence of each geographical market independently, the day-light saving time, the local holidays and the peaks due to news announcements documented in Guillaume et al. (1994a). Only some stochastic seasonality can be seen in the autocorrelation function of the volatility in $\vartheta$-time shown in Figure 2(b).

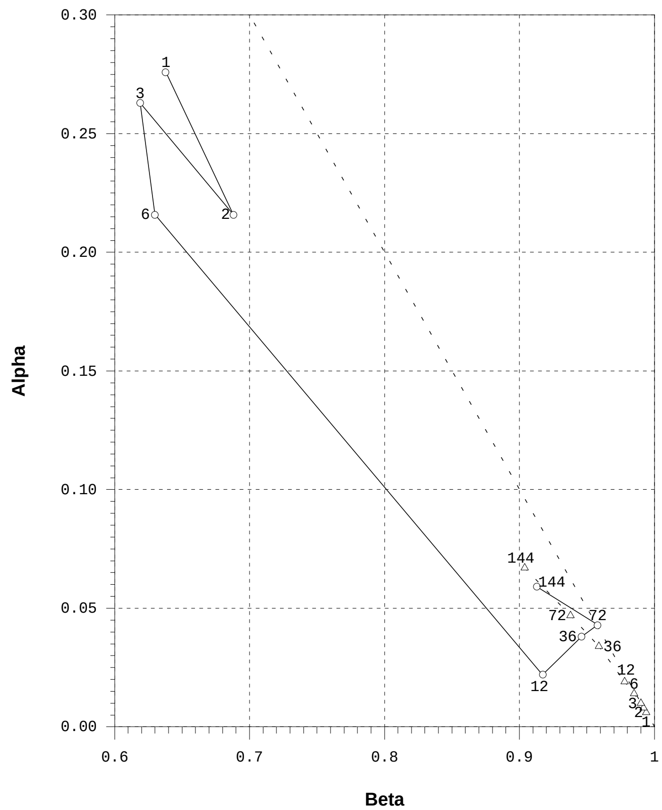
## 4 Results of the GARCH coefficient estimates

In order to test the effects of the geographical and temporal heterogeneity of the markets, we used the Maximum Likelihood procedure to fit the simple following GARCH(1,1) on the time series for several frequencies and time scales:

$$h(t) = \alpha_0 + \alpha_1 . \varepsilon^2(t-1) + \beta_1 . h(t-1) \tag{4.1}$$

where $h(t)$ is the conditional variance and $\varepsilon^2(t)$ is the squared innovation. At the highest frequencies (less than 2 hours), we included a MA(1) term in the mean equation to account for the statistically significant auto-correlation of the returns at these frequencies.

The algorithm used for the numerical optimization of the Maximum Likelihood is a two-steps iterative procedure that combines a Genetic Algorithm (GA) with the Berndt, Hall, Hall and Hausman (BHHH) algorithm (Berndt et al., 1974). In this algorithm, the number of iterations – called generations – is fixed. For each generation, the GA initializes a fixed number of potential solutions for the parameters (Goldberg, 1989). These solutions might either be chosen randomly or be picked up from a given set of initial solutions (for example, solutions for similar problems). In this paper, we chose the random initialization to avoid any a priori bias in the estimation. The Maximum Likelihood is then evaluated for each potential solution. In the second step, the best solution is used as an initial solution for the BHHH algorithm. The solution obtained with



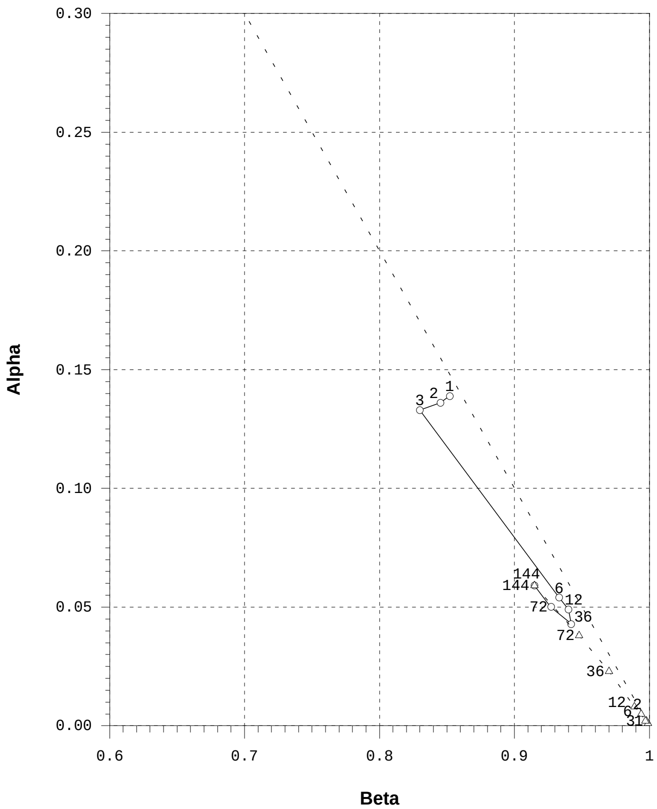

Aggregation of the GARCH(1,1) for estimated coefficients in business time (o) and theoretically derived coefficients (△) using the Nelson results for the USD/FRF for different aggregation factors (1 = 10 m; 2 = 20 m; 3 = 30 m; 6 = 1 h; 12 = 2 h; 36 = 6 h; 72 = 12 h; 144= 24 h). The diagonal dotted line represents the limit for which $\alpha + \beta = 1$.

Figure 3: Aggregation of the GARCH(1,1) coefficients in physical time.

the BHHH algorithm is added to the set of random potential solutions for the next generation. Convergence is ensured by a sufficient number of generations. The main advantage of this method is to avoid to be trapped in local minima with the BHHH algorithm, as the set of potential solutions for the next generation of the GA includes randomly chosen solutions. The method is also very fast, notwithstanding the very large number of observations (368.000 data points for the 10 minutes frequency). Robust standard errors were computed using White's variance-covariance matrix (White, 1980).

## 4.1 Impact of the seasonal heterogeneity

As could be expected, the use of the physical time scale caused a complete break-down of the estimation procedure yielding $\alpha_1$ estimates as large as 4.0. This is, of course, due to the presence of the week-ends. We, therefore, re-estimate the model in the previously defined business time scale. The circles in Figure 3 correspond to the estimation of the $\alpha_1$ and $\beta_1$ coefficients for the

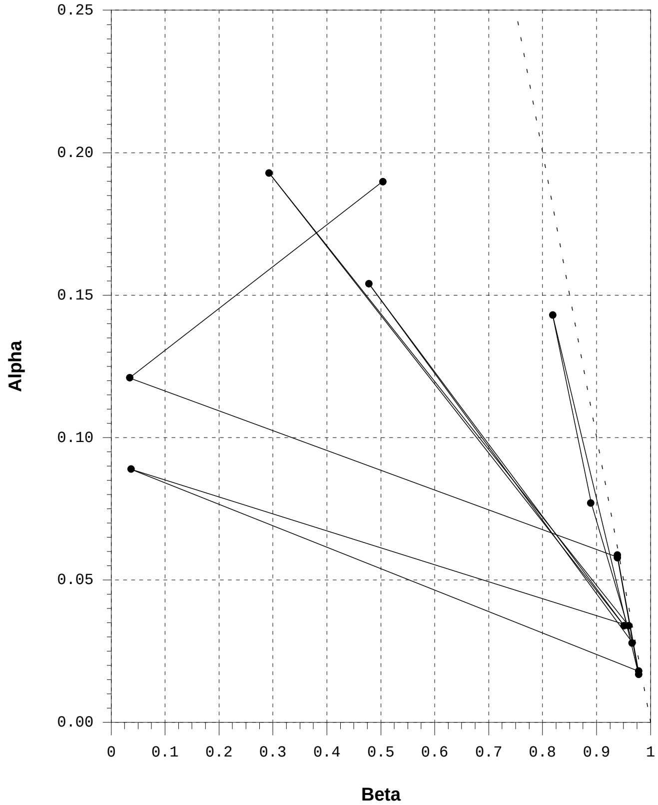



Aggregation of the GARCH(1,1) for estimated coefficients in $\vartheta$-time (o) and theoretically derived coefficients ($\triangle$) using the Drost and Nijman results for the USD/DEM for different aggregation factors (1 = 10 m; 2 = 20 m; 3 = 30 m; 6 = 1 h; 12 = 2 h; 36 = 6 h; 72 = 12 h; 144= 24 h). The diagonal dotted line represents the limit for which $\alpha + \beta = 1$.

Figure 4: Aggregation of the GARCH(1,1) coefficients in $\vartheta$-time.

USD/FRF for several frequencies. Although the coefficient estimates may look quite reasonable at one particular frequency, the global picture for all the frequencies appears quite odd. In particular, the coefficient estimates for frequencies higher than the 2 hours time interval reflect the relatively random behavior of FX returns at these frequencies.

Therefore, we re-estimated the model once again using the $\vartheta$-time scale to account for the geographical heterogeneity of the markets. The results for the USD/DEM are represented in Figure 4 by circles. More detailed results for each currency can also be found in Tables 2 and 3 in the Appendix. This time, contrary to results in business time, coefficient estimates are quite similar across FX rates and reasonably homogeneous from one frequency to another (see Andersen and Bollerslev (1994) for similar results). This indicates the importance of taking this seasonal heterogeneity in empirical studies of the FX markets such as lead-lag relationships between markets. However, this does not imply that the GARCH model describes the volatility process at the intra-daily frequencies as adequately as it seems to do at the daily or weekly frequencies. One




The temporal stability of the GARCH(1,1) coefficients for sub-periods of six months for the USD/DEM at the 2 hours frequency. The time scale is the $\vartheta$-time.

Figure 5: Temporal stability of the GARCH(1,1) coefficient estimates.

easy test is to check whether the temporal aggregation properties of the GARCH model hold at the intra-daily frequencies.

## 4.2 Impact of the temporal heterogeneity

The GARCH model can be viewed as describing either a jump process (Drost and Nijman, 1993) or a diffusion process (Nelson and Foster, 1994). In both cases, the sum of the $\alpha_1$ and $\beta_1$ should tend to 1 as the frequency becomes larger and larger and the autoregressive parameter $\beta_1$ should tend to 1, whereas the moving average parameter $\alpha_1$ should tend to 0. In other words, the higher the frequency, the longer the effect of shocks on the volatility should persist. The triangles in Figures 3 and 4 represent the theoretically derived coefficients on the basis of the formulae presented in Nelson (1990) and in Drost and Nijman (1993), respectively.

Since previous results confirmed the adequacy of these theoretical results at the daily and weekly frequencies (Drost and Nijman, 1993), we used the daily estimations as a starting point to compute the results for the other frequencies. As can also be checked in Tables 2 and 3 in the Appendix, both formulas yield similar results. Although the coefficient estimates in the case of the $\vartheta$-time scale seem quite reasonable, they are outside the significance interval of the coefficients that are derived theoretically. Moreover, the $\alpha_1$'s increase slowly whereas the $\beta_1$'s decrease and the sum of the two parameters does not really tend to 1 as one goes from the daily to higher frequencies. Besides, at frequencies higher than 2 hours, the sum of the two parameters start to decrease, implying even less volatility persistence. As already noted in the case of the business time scale, this reflects the seemingly more random behavior of FX returns at these frequencies. This probably also explains the abnormal results we obtained at these frequencies for the fit of the GARCH model using a T-Student distribution instead of the normal distribution such as in Baillie and Bollerslev (1989)[1].

Therefore, even when one takes into account the impact of the geographical heterogeneity of the traders, the GARCH model does not appear to be able to capture adequately the heterogeneity of traders acting at different time-horizons. In other words, the information content of the conditional density is not the same for different frequencies. This could be due to different volatilities being relevant at different frequencies, or the presence of traders with different objective functions acting at different time-horizons.

To further assess the time heterogeneous behavior of returns, we looked at the temporal stability of the coefficient estimates for several sub-samples. Figure 5 gives the estimations of the GARCH parameters for the USD/DEM at the 2 hours time-interval, for sub-samples of six months, yielding 2,190 observations per sub-sample in $\vartheta$-time[2]. As can be seen, the the coefficients are not stable over time. Estimation of the Likelihood Ratio test of the equality of the parameters across the sub-samples yields a value of 7,300 compared to the corresponding critical value of 55.5 at the 0.001 level. Moreover, the absence of pattern in their evolution reflects the presence of a very long term, non-GARCH, component in FX rates.

### 4.3 Out-of-sample forecasting

A very interesting application of the GARCH model is its potential use for the forecast of the volatility such as in the context of option pricing models. To test the out-of-sample performance of the GARCH(1,1), we fitted the GARCH model on the first half of the sample and used the second half of the sample to compute the forecast. The one step ahead forecast of the conditional volatility for the GARCH is compared with the corresponding (unconditional) historical volatility computed as the squared of the innovations. The expected conditional variance for the GARCH is given by the following formula (Engle and Bollerslev, 1986):

$$E_t(h(t+1)) \;=\; \alpha_0 \;+\; \alpha_1.\varepsilon^2(t) \;+\; \beta_1.h(t) \tag{4.2}$$

for the one-step ahead conditional forecast.

To account for the various sources of heterogeneity, we used the $\vartheta$-time scale for several frequencies. As an alternative forecast, we used the historical volatility of the previous period.

Tables 4, 5 and 8 give the Root Mean Squared Errors (RMSE) for the different frequencies and, in the case of the USD/DEM at the 2-hours frequency, for sub-samples of six months. In all cases, the RMSE for the historical volatility is found to be much lower for the historical volatility. In order to test formally the efficiency of our forecasts, we then regressed the one-step ahead historical volatility on the one-step ahead forecast, that is, we estimate the following equation in the case of the GARCH model:

$$\varepsilon^2(t+1) \;=\; b_0 \;+\; b_1.E_t(h(t+1)) \;+\; \mu_{t+1} \tag{4.3}$$

[1] Although the algorithm converges, the sum of the $\alpha_1$ and $\beta_1$ increasingly exceeds 1 as the frequency becomes larger. One also finds excess residual skewness and kurtosis. Since these results are robust to the size of the sample, they cannot be attributed to a larger number of observations in the tails

[2] Tables with coefficient estimates are available upon request.

where $b_0$ should be equal to zero and $b_1$ should be statistically equal to one. To avoid auto-correlation and heteroskedasticity problems, we used the procedure described in Newey and West (1987) to estimate robust standard errors. Note that in the hypothesis that the $\mu_t$ are not auto-correlated, the results are not biased. These results are given in Table 6, 6  and 9. Both in the case of the GARCH forecast and the historical volatility, the regression parameters $b_1$ are very significantly different from one and the corresponding $R^2$'s are extremely low, showing that neither of the forecasts are very accurate. These results only confirm previous findings with daily data (Day and Lewis, 1992; West and Cho, 1994), reinforcing the empirical evidence of the low predictive power of the GARCH model for the volatility.

# 5 Conclusion

In this paper, we assessed the impact of the heterogeneous behaviour of the FX markets on the intra-daily performance of the GARCH model. We found that the various sources of deterministic seasonality present in intra-daily returns introduced severe mis-specifications. However, we showed that these inefficiencies could best be dealt with by the use of a time-scale transformation. Though, even in this new time-scale, the aggregation properties of the GARCH break down at the intra-daily frequencies, revealing the presence of several components corresponding to different time-horizons. Although the explicit modelling of these different time-components remains a challenge[3], a second-best solution would be to vary the sampling frequency of the data used in micro-structure studies.

However, in addition to this seasonal and temporal heterogeneity, two distinct patterns at both extremes of the frequency spectrum were highlighted. On the one hand, at frequencies higher than 2 hours, the effects of the price formation process taking place at the 10 minutes frequency were found to cause a lower persistence of the volatility. On the other hand, we could also identify the presence of a very long-term, seemingly random, component at intervals of several months. This long-term component is consistent with the observed changing views on the relevant underlying fundamentals in the long run. Furthermore, even at frequencies in the middle range between these two extremes, the relatively poor out-of-sample forecasting power of the GARCH model indicated that other sources of heterogeneity are yet to be identified and modelled. Therefore, it remains to be seen whether these additional sources of heterogeneity could be captured by a single process equation such as a (very) sophisticated GARCH-type model.

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[3] See Müller et al. (1994) for a first attempt.

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# 6 Appendix: Tables of estimates

| rate | time int. | mean | variance | skewness | kurtosis |
|---|---|---|---|---|---|
| USD/DEM | 10 m | $-2.73 \cdot 10^{-7}$ | $2.62 \cdot 10^{-7}$ | 0.17 | 35.10 |
| | 1 h | $-1.63 \cdot 10^{-6}$ | $1.45 \cdot 10^{-6}$ | 0.26 | 23.55 |
| | 6 h | $-9.84 \cdot 10^{-6}$ | $9.20 \cdot 10^{-6}$ | 0.24 | 9.44 |
| | 24 h | $-4.00 \cdot 10^{-5}$ | $3.81 \cdot 10^{-5}$ | 0.08 | 3.33 |
| | 1 w | $-2.97 \cdot 10^{-4}$ | $2.64 \cdot 10^{-4}$ | 0.18 | 0.71 |
| USD/JPY | 10 m | $-9.42 \cdot 10^{-7}$ | $2.27 \cdot 10^{-7}$ | $-0.18$ | 26.40 |
| | 1 h | $-5.67 \cdot 10^{-6}$ | $1.27 \cdot 10^{-6}$ | $-0.09$ | 25.16 |
| | 6 h | $-3.40 \cdot 10^{-5}$ | $7.63 \cdot 10^{-6}$ | $-0.05$ | 11.65 |
| | 24 h | $-1.37 \cdot 10^{-4}$ | $3.07 \cdot 10^{-5}$ | $-0.15$ | 4.81 |
| | 1 w | $-9.61 \cdot 10^{-4}$ | $2.27 \cdot 10^{-4}$ | $-0.27$ | 1.30 |
| GBP/USD | 10 m | $-6.91 \cdot 10^{-9}$ | $2.38 \cdot 10^{-7}$ | 0.02 | 27.46 |
| | 1 h | $7.61 \cdot 10^{-7}$ | $1.40 \cdot 10^{-6}$ | $-0.23$ | 21.53 |
| | 6 h | $4.63 \cdot 10^{-6}$ | $8.85 \cdot 10^{-6}$ | $-0.34$ | 10.09 |
| | 24 h | $1.72 \cdot 10^{-5}$ | $3.60 \cdot 10^{-5}$ | $-0.26$ | 4.41 |
| | 1 w | $6.99 \cdot 10^{-5}$ | $2.72 \cdot 10^{-4}$ | $-0.66$ | 2.77 |
| USD/CHF | 10 m | $-2.28 \cdot 10^{-7}$ | $3.07 \cdot 10^{-7}$ | $-0.04$ | 23.85 |
| | 1 h | $-1.37 \cdot 10^{-6}$ | $1.75 \cdot 10^{-6}$ | 0.05 | 18.28 |
| | 6 h | $-8.23 \cdot 10^{-6}$ | $1.11 \cdot 10^{-5}$ | 0.05 | 7.73 |
| | 24 h | $-3.38 \cdot 10^{-5}$ | $4.51 \cdot 10^{-5}$ | $-0.04$ | 2.81 |
| | 1 w | $-2.58 \cdot 10^{-4}$ | $3.16 \cdot 10^{-4}$ | 0.09 | 0.34 |
| USD/FRF | 10 m | $-1.98 \cdot 10^{-7}$ | $2.08 \cdot 10^{-7}$ | 0.35 | 43.31 |
| | 1 h | $-1.18 \cdot 10^{-6}$ | $1.28 \cdot 10^{-6}$ | 0.47 | 28.35 |
| | 6 h | $-7.13 \cdot 10^{-6}$ | $8.29 \cdot 10^{-6}$ | 0.23 | 9.69 |
| | 24 h | $-2.91 \cdot 10^{-5}$ | $3.40 \cdot 10^{-5}$ | 0.06 | 3.22 |
| | 1 w | $-2.32 \cdot 10^{-4}$ | $2.44 \cdot 10^{-4}$ | 0.16 | 0.88 |

Table 1: Price change distributions characterized by the mean, the variance, the skewness and the kurtosis at different time int.s for the major currencies against the USD

| rate | time int. | M.L. | $\alpha_o$ | $\alpha_1$ | $\beta_1$ | $\alpha_1 + \beta_1$ | K | $\alpha_1'$ | $\beta_1'$ | $\alpha_1' + \beta_1'$ | K' | $\alpha_1''$ | $\beta_1''$ | $\alpha_1'' + \beta_1''$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| USD/DEM | 10 m | - 0.61076 | $0.57 \cdot 10^{-6}$ | 0.138 | 0.858 | 0.996 | 22.46 | 0.001 | 0.999 | 1.000 | 294.11 | 0.005 | 0.995 | 1.000 |
| | | | $(0.61 \cdot 10^{-7})$ pr | (0.0092) | (0.0080) | | | | | | | | | |
| | 20 m | - 0.97428 | $0.29 \cdot 10^{-5}$ | 0.164 | 0.788 | 0.952 | 47.33 | 0.002 | 0.998 | 1.000 | 97.37 | 0.007 | 0.993 | 1.000 |
| | | | $(0.12 \cdot 10^{-5})$ | (0.0266) | (0.0536) | | | | | | | | | |
| | 30 m | - 1.19167 | $0.35 \cdot 10^{-5}$ | 0.125 | 0.840 | 0.965 | 37.25 | 0.002 | 0.997 | 0.999 | 65.13 | 0.009 | 0.991 | 1.000 |
| | | | $(0.94 \cdot 10^{-6})$ | (0.0225) | (0.0286) | | | | | | | | | |
| | 1 h | - 1.54224 | $0.18 \cdot 10^{-5}$ | 0.044 | 0.945 | 0.989 | 16.51 | 0.005 | 0.994 | 0.999 | 32.88 | 0.012 | 0.987 | 0.999 |
| | | | $(0.14 \cdot 10^{-5})$ | (0.0190) | (0.0277) | | | | | | | | | |
| | 2 h | - 1.89236 | $0.41 \cdot 10^{-4}$ | 0.049 | 0.940 | 0.989 | 10.87 | 0.008 | 0.989 | 0.997 | 16.77 | 0.017 | 0.980 | 0.997 |
| | | | $(0.18 \cdot 10^{-5})$ | (0.0016) | (0.0017) | | | | | | | | | |
| | 6 h | - 2.44953 | $0.14 \cdot 10^{-3}$ | 0.043 | 0.942 | 0.985 | 6.04 | 0.023 | 0.970 | 0.993 | 6.07 | 0.030 | 0.964 | 0.994 |
| | | | $(0.16 \cdot 10^{-4})$ | (0.0033) | (0.0041) | | | | | | | | | |
| | 12 h | - 0.51399 | $0.43 \cdot 10^{-4}$ | 0.050 | 0.927 | 0.977 | 3.97 | 0.038 | 0.948 | 0.986 | 3.45 | 0.042 | 0.945 | 0.987 |
| | | | $(0.84 \cdot 10^{-4})$ | (0.0077) | (0.0098) | | | | | | | | | |
| | 24 h | - 0.89680 | $0.10 \cdot 10^{-3}$ | 0.059 | 0.915 | 0.974 | 2.21 | | | | | | | |
| | | | $(0.41 \cdot 10^{-4})$ | (0.0133) | (0.0199) | | | | | | | | | |
| GBP/USD | 10 m | - 0.58114 | $0.10 \cdot 10^{-4}$ | 0.160 | 0.814 | 0.974 | 20.18 | 0.001 | 0.999 | 1.000 | 343.20 | 0.004 | 0.996 | 1.000 |
| | | | $(0.94 \cdot 10^{-7})$ | (0.0100) | (0.0108) | | | | | | | | | |
| | 20 m | - 0.95952 | $0.29 \cdot 10^{-5}$ | 0.153 | 0.803 | 0.956 | 34.42 | 0.001 | 0.998 | 0.999 | 173.51 | 0.006 | 0.994 | 1.000 |
| | | | $(0.37 \cdot 10^{-6})$ | (0.0124) | (0.0165) | | | | | | | | | |
| | 30 m | - 1.17450 | $0.55 \cdot 10^{-5}$ | 0.153 | 0.783 | 0.936 | 16.21 | 0.002 | 0.998 | 1.000 | 115.95 | 0.008 | 0.992 | 1.000 |
| | | | $(0.10 \cdot 10^{-5})$ | (0.0190) | (0.0275) | | | | | | | | | |
| | 1 h | - 1.54466 | $0.26 \cdot 10^{-5}$ | 0.046 | 0.938 | 0.984 | 17.59 | 0.003 | 0.995 | 0.998 | 58.39 | 0.010 | 0.988 | 0.998 |
| | | | $(0.16 \cdot 10^{-5})$ | (0.0174) | (0.0280) | | | | | | | | | |
| | 2 h | - 1.89712 | $0.67 \cdot 10^{-4}$ | 0.050 | 0.928 | 0.978 | 9.76 | 0.007 | 0.990 | 0.997 | 29.62 | 0.015 | 0.983 | 0.998 |
| | | | $(0.37 \cdot 10^{-5})$ | (0.0020) | (0.0028) | | | | | | | | | |
| | 6 h | - 2.45888 | $0.19 \cdot 10^{-3}$ | 0.044 | 0.934 | 0.978 | 4.60 | 0.018 | 0.973 | 0.991 | 10.46 | 0.026 | 0.967 | 0.993 |
| | | | $(0.24 \cdot 10^{-4})$ | (0.0040) | (0.0059) | | | | | | | | | |
| | 12 h | - 0.53065 | $0.49 \cdot 10^{-4}$ | 0.043 | 0.930 | 0.973 | 4.16 | 0.032 | 0.952 | 0.984 | 5.71 | 0.036 | 0.947 | 0.983 |
| | | | $(0.99 \cdot 10^{-5})$ | (0.0063) | (0.0102) | | | | | | | | | |
| | 24 h | - 0.91185 | $0.13 \cdot 10^{-3}$ | 0.051 | 0.916 | 0.967 | 3.39 | | | | | | | |
| | | | $(0.46 \cdot 10^{-4})$ | (0.0072) | (0.0164) | | | | | | | | | |
| USD/JPY | 10 m | - 0.54555 | $0.47 \cdot 10^{-6}$ | 0.118 | 0.873 | 0.991 | 25.14 | 0.001 | 0.999 | 1.000 | 190.55 | 0.006 | 0.994 | 1.000 |
| | | | $(0.69 \cdot 10^{-7})$ | (0.0074) | (0.0073) | | | | | | | | | |
| | 20 m | - 0.87865 | $0.11 \cdot 10^{-5}$ | 0.123 | 0.860 | 0.983 | 17.95 | 0.002 | 0.997 | 0.999 | 96.68 | 0.008 | 0.991 | 0.999 |
| | | | $(0.21 \cdot 10^{-6})$ | (0.0099) | (0.0113) | | | | | | | | | |
| | 30 m | - 1.08588 | $0.18 \cdot 10^{-5}$ | 0.118 | 0.865 | 0.983 | 16.65 | 0.003 | 0.996 | 0.999 | 64.06 | 0.010 | 0.989 | 0.999 |
| | | | $(0.37 \cdot 10^{-6})$ | (0.0138) | (0.0161) | | | | | | | | | |
| | 1 h | - 1.43892 | $0.31 \cdot 10^{-5}$ | 0.091 | 0.890 | 0.981 | 16.10 | 0.006 | 0.992 | 0.998 | 32.45 | 0.015 | 0.984 | 0.999 |
| | | | $(0.14 \cdot 10^{-5})$ | (0.0207) | (0.0278) | | | | | | | | | |
| | 2 h | - 0.01082 | $0.63 \cdot 10^{-4}$ | 0.078 | 0.901 | 0.979 | 11.16 | 0.012 | 0.985 | 0.997 | 16.66 | 0.020 | 0.976 | 0.996 |
| | | | $(0.25 \cdot 10^{-4})$ | (0.0024) | (0.0027) | | | | | | | | | |
| | 6 h | - 2.36557 | $0.16 \cdot 10^{-3}$ | 0.057 | 0.923 | 0.980 | 4.63 | 0.029 | 0.959 | 0.988 | 6.18 | 0.036 | 0.953 | 0.989 |
| | | | $(0.18 \cdot 10^{-4})$ | (0.0046) | (0.0057) | | | | | | | | | |
| | 12 h | - 0.40718 | $0.47 \cdot 10^{-4}$ | 0.072 | 0.898 | 0.970 | 3.13 | 0.048 | 0.929 | 0.977 | 3.62 | 0.050 | 0.927 | 0.977 |
| | | | $(0.96 \cdot 10^{-4})$ | (0.0100) | (0.0138) | | | | | | | | | |
| | 24 h | - 0.78453 | $0.14 \cdot 10^{-3}$ | 0.072 | 0.884 | 0.956 | 2.44 | | | | | | | |
| | | | $(0.46 \cdot 10^{-4})$ | (0.0163) | (0.0262) | | | | | | | | | |

Table 2: Parameters estimates for the GARCH(1,1) with standardised maximum likelihood (M.L.) and kurtosis (K). Corrected standard errors are in parenthesis. The time scale is the deseasonalised $\vartheta$-time. The coefficients with a prime and and a double prime are computed from the (dis)aggregation formulas for the jump and the diffusion hypothesis respectively. The daily interval serves as a reference basis.

| rate | time int. | M.L. | $\alpha_o$ | $\alpha_1$ | $\beta_1$ | $\alpha_1+\beta_1$ | K | $\alpha_1'$ | $\beta_1'$ | $\alpha_1'+\beta_1'$ | K' | $\alpha_1''$ | $\beta_1''$ | $\alpha_1''+\beta_1''$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| USD/CHF | 10 m | - 0.71958 | $0.86 \cdot 10^{-6}$ | 0.130 | 0.854 | 0.984 | 12.84 | 0.001 | 0.999 | 1.000 | 146.79 | 0.004 | 0.996 | 1.000 |
| | | | $(0.97 \cdot 10^{-7})$ | (0.0084) | (0.0092) | | | | | | | | | |
| | 20 m | - 1.07187 | $0.22 \cdot 10^{-4}$ | 0.129 | 0.845 | 0.974 | 10.96 | 0.001 | 0.998 | 0.999 | 73.56 | 0.006 | 0.994 | 1.000 |
| | | | $(0.30 \cdot 10^{-6})$ | (0.0105) | (0.0128) | | | | | | | | | |
| | 30 m | - 1.29173 | $0.49 \cdot 10^{-5}$ | 0.140 | 0.817 | 0.957 | 10.98 | 0.002 | 0.997 | 0.999 | 49.19 | 0.007 | 0.993 | 1.000 |
| | | | $(0.87 \cdot 10^{-6})$ | (0.0162) | (0.0213) | | | | | | | | | |
| | 1 h | - 1.65621 | $0.53 \cdot 10^{-4}$ | 0.067 | 0.905 | 0.972 | 9.60 | 0.004 | 0.994 | 0.998 | 26.82 | 0.010 | 0.989 | 0.999 |
| | | | $(0.35 \cdot 10^{-5})$ | (0.0274) | (0.0446) | | | | | | | | | |
| | 2 h | - 2.00321 | $0.87 \cdot 10^{-4}$ | 0.054 | 0.923 | 0.977 | 9.46 | 0.008 | 0.989 | 0.997 | 12.64 | 0.014 | 0.983 | 0.997 |
| | | | $(0.52 \cdot 10^{-5})$ | (0.0022) | (0.0032) | | | | | | | | | |
| | 6 h | - 0.25840 | $0.31 \cdot 10^{-4}$ | 0.051 | 0.921 | 0.972 | 4.32 | 0.020 | 0.972 | 0.992 | 4.55 | 0.025 | 0.968 | 0.993 |
| | | | $(0.28 \cdot 10^{-5})$ | (0.0037) | (0.0046) | | | | | | | | | |
| | 12 h | - 0.62878 | $0.52 \cdot 10^{-4}$ | 0.047 | 0.930 | 0.977 | 2.30 | 0.033 | 0.952 | 0.985 | 2.57 | 0.034 | 0.950 | 0.984 |
| | | | $(0.12 \cdot 10^{-4})$ | (0.0076) | (0.0106) | | | | | | | | | |
| | 24 h | - 0.98481 | $0.13 \cdot 10^{-3}$ | 0.049 | 0.922 | 0.971 | 1.62 | | | | | | | |
| | | | $(0.64 \cdot 10^{-4})$ | (0.0139) | (0.0243) | | | | | | | | | |
| USD/FRF | 10 m | - 0.50032 | $0.50 \cdot 10^{-5}$ | 0.144 | 0.848 | 0.992 | 17.26 | 0.001 | 0.999 | 1.000 | 126.27 | 0.006 | 0.994 | 1.000 |
| | | | $(0.45 \cdot 10^{-7})$ | (0.0089) | (0.0083) | | | | | | | | | |
| | 20 m | - 0.88450 | $0.15 \cdot 10^{-4}$ | 0.162 | 0.826 | 0.988 | 26.58 | 0.002 | 0.997 | 0.999 | 62.99 | 0.008 | 0.992 | 1.000 |
| | | | $(0.25 \cdot 10^{-6})$ | (0.0238) | (0.0211) | | | | | | | | | |
| | 30 m | - 1.10742 | $0.33 \cdot 10^{-4}$ | 0.168 | 0.801 | 0.969 | 18.72 | 0.003 | 0.996 | 0.999 | 42.27 | 0.010 | 0.990 | 1.000 |
| | | | $(0.57 \cdot 10^{-6})$ | (0.0239) | (0.0242) | | | | | | | | | |
| | 1 h | - 1.48620 | $0.41 \cdot 10^{-4}$ | 0.087 | 0.888 | 0.975 | 17.30 | 0.006 | 0.993 | 0.999 | 21.34 | 0.014 | 0.985 | 0.999 |
| | | | $(0.5 \cdot 10^{-7})$ | (0.0010) | (0.0011) | | | | | | | | | |
| | 2 h | - 1.84558 | $0.36 \cdot 10^{-4}$ | 0.048 | 0.941 | 0.989 | 12.35 | 0.011 | 0.986 | 0.997 | 6.97 | 0.019 | 0.978 | 0.997 |
| | | | $(0.18 \cdot 10^{-5})$ | (0.0016) | (0.0016) | | | | | | | | | |
| | 6 h | - 2.41546 | $0.98 \cdot 10^{-4}$ | 0.043 | 0.946 | 0.989 | 5.77 | 0.028 | 0.964 | 0.993 | 4.10 | 0.034 | 0.959 | 0.993 |
| | | | $(0.11 \cdot 10^{-4})$ | (0.0027) | (0.0027) | | | | | | | | | |
| | 12 h | - 0.47338 | $0.41 \cdot 10^{-4}$ | 0.056 | 0.920 | 0.976 | 2.79 | 0.046 | 0.940 | 0.986 | 2.45 | 0.047 | 0.938 | 0.986 |
| | | | $(0.82 \cdot 10^{-5})$ | (0.0078) | (0.0104) | | | | | | | | | |
| | 24 h | - 0.83423 | $0.10 \cdot 10^{-3}$ | 0.067 | 0.904 | 0.971 | 1.71 | | | | | | | |
| | | | $(0.35 \cdot 10^{-4})$ | (0.0150) | (0.0205) | | | | | | | | | |

Table 3: Parameters estimates for the GARCH(1,1) with standardised maximum likelihood (M.L.) and kurtosis (K). C
dard errors are in parenthesis. The time scale is the de-seasonalised $\vartheta$-time. The coefficients with a prime an
prime are computed from the (dis)aggregation formulas for the jump and the diffusion hypotheses respecti
interval serves as a reference basis.

| rate | time int. | G(1) | G(2) | G(12) | G(24) | HV(1) | HV(2) | HV(12) | HV(24) |
|---|---|---|---|---|---|---|---|---|---|
| USD/DEM | 10 m | $0.43 \cdot 10^{-4}$ | $0.49 \cdot 10^{-4}$ | $0.10 \cdot 10^{-3}$ | $0.17 \cdot 10^{-3}$ | $0.36 \cdot 10^{-5}$ | $0.48 \cdot 10^{-5}$ | $0.48 \cdot 10^{-5}$ | $0.48 \cdot 10^{-5}$ |
| | 20 m | $0.12 \cdot 10^{-3}$ | $0.13 \cdot 10^{-3}$ | $0.27 \cdot 10^{-3}$ | $0.41 \cdot 10^{-3}$ | $0.53 \cdot 10^{-5}$ | $0.72 \cdot 10^{-5}$ | $0.73 \cdot 10^{-5}$ | $0.73 \cdot 10^{-5}$ |
| | 30 m | $0.22 \cdot 10^{-3}$ | $0.25 \cdot 10^{-3}$ | $0.50 \cdot 10^{-3}$ | $0.69 \cdot 10^{-3}$ | $0.68 \cdot 10^{-5}$ | $0.92 \cdot 10^{-5}$ | $0.93 \cdot 10^{-5}$ | $0.93 \cdot 10^{-5}$ |
| | 1 h | $0.36 \cdot 10^{-3}$ | $0.38 \cdot 10^{-3}$ | $0.56 \cdot 10^{-3}$ | $0.74 \cdot 10^{-3}$ | $0.10 \cdot 10^{-4}$ | $0.14 \cdot 10^{-4}$ | $0.14 \cdot 10^{-4}$ | $0.14 \cdot 10^{-4}$ |
| | 2 h | $0.68 \cdot 10^{-3}$ | $0.72 \cdot 10^{-3}$ | $0.10 \cdot 10^{-2}$ | $0.14 \cdot 10^{-2}$ | $0.17 \cdot 10^{-4}$ | $0.23 \cdot 10^{-4}$ | $0.23 \cdot 10^{-4}$ | $0.23 \cdot 10^{-4}$ |
| | 6 h | $0.24 \cdot 10^{-2}$ | $0.25 \cdot 10^{-2}$ | $0.35 \cdot 10^{-2}$ | $0.44 \cdot 10^{-2}$ | $0.33 \cdot 10^{-4}$ | $0.43 \cdot 10^{-4}$ | $0.43 \cdot 10^{-4}$ | $0.44 \cdot 10^{-4}$ |
| | 12 h | $0.58 \cdot 10^{-3}$ | $0.61 \cdot 10^{-3}$ | $0.87 \cdot 10^{-3}$ | $0.11 \cdot 10^{-2}$ | $0.55 \cdot 10^{-4}$ | $0.71 \cdot 10^{-4}$ | $0.71 \cdot 10^{-4}$ | $0.72 \cdot 10^{-4}$ |
| | 24 h | $0.12 \cdot 10^{-2}$ | $0.12 \cdot 10^{-2}$ | $0.18 \cdot 10^{-2}$ | $0.24 \cdot 10^{-2}$ | $0.10 \cdot 10^{-3}$ | $0.14 \cdot 10^{-3}$ | $0.14 \cdot 10^{-3}$ | $0.14 \cdot 10^{-3}$ |
| USD/GBP | 10 m | $0.56 \cdot 10^{-4}$ | $0.65 \cdot 10^{-4}$ | $0.14 \cdot 10^{-3}$ | $0.21 \cdot 10^{-3}$ | $0.14 \cdot 10^{-5}$ | $0.18 \cdot 10^{-5}$ | $0.20 \cdot 10^{-5}$ | $0.20 \cdot 10^{-5}$ |
| | 20 m | $0.14 \cdot 10^{-3}$ | $0.17 \cdot 10^{-3}$ | $0.34 \cdot 10^{-3}$ | $0.47 \cdot 10^{-3}$ | $0.29 \cdot 10^{-5}$ | $0.39 \cdot 10^{-5}$ | $0.40 \cdot 10^{-5}$ | $0.40 \cdot 10^{-5}$ |
| | 30 m | $0.25 \cdot 10^{-3}$ | $0.29 \cdot 10^{-3}$ | $0.57 \cdot 10^{-3}$ | $0.73 \cdot 10^{-3}$ | $0.41 \cdot 10^{-5}$ | $0.54 \cdot 10^{-5}$ | $0.56 \cdot 10^{-5}$ | $0.56 \cdot 10^{-5}$ |
| | 1 h | $0.47 \cdot 10^{-3}$ | $0.50 \cdot 10^{-3}$ | $0.75 \cdot 10^{-3}$ | $0.97 \cdot 10^{-3}$ | $0.77 \cdot 10^{-5}$ | $0.10 \cdot 10^{-4}$ | $0.10 \cdot 10^{-4}$ | $0.10 \cdot 10^{-4}$ |
| | 2 h | $0.93 \cdot 10^{-3}$ | $0.98 \cdot 10^{-3}$ | $0.14 \cdot 10^{-2}$ | $0.18 \cdot 10^{-2}$ | $0.13 \cdot 10^{-4}$ | $0.18 \cdot 10^{-4}$ | $0.18 \cdot 10^{-4}$ | $0.18 \cdot 10^{-4}$ |
| | 6 h | $0.29 \cdot 10^{-2}$ | $0.30 \cdot 10^{-2}$ | $0.41 \cdot 10^{-2}$ | $0.52 \cdot 10^{-2}$ | $0.30 \cdot 10^{-4}$ | $0.39 \cdot 10^{-4}$ | $0.40 \cdot 10^{-4}$ | $0.40 \cdot 10^{-4}$ |
| | 12 h | $0.69 \cdot 10^{-3}$ | $0.72 \cdot 10^{-3}$ | $0.98 \cdot 10^{-3}$ | $0.12 \cdot 10^{-2}$ | $0.59 \cdot 10^{-4}$ | $0.77 \cdot 10^{-4}$ | $0.79 \cdot 10^{-4}$ | $0.79 \cdot 10^{-4}$ |
| | 24 h | $0.15 \cdot 10^{-2}$ | $0.16 \cdot 10^{-2}$ | $0.23 \cdot 10^{-2}$ | $0.28 \cdot 10^{-2}$ | $0.15 \cdot 10^{-3}$ | $0.20 \cdot 10^{-3}$ | $0.21 \cdot 10^{-3}$ | $0.21 \cdot 10^{-3}$ |
| JPY/USD | 10 m | $0.39 \cdot 10^{-4}$ | $0.45 \cdot 10^{-4}$ | $0.97 \cdot 10^{-4}$ | $0.15 \cdot 10^{-3}$ | $0.11 \cdot 10^{-5}$ | $0.15 \cdot 10^{-5}$ | $0.16 \cdot 10^{-5}$ | $0.16 \cdot 10^{-5}$ |
| | 20 m | $0.79 \cdot 10^{-4}$ | $0.90 \cdot 10^{-4}$ | $0.19 \cdot 10^{-3}$ | $0.29 \cdot 10^{-3}$ | $0.20 \cdot 10^{-5}$ | $0.26 \cdot 10^{-5}$ | $0.27 \cdot 10^{-5}$ | $0.27 \cdot 10^{-5}$ |
| | 30 m | $0.13 \cdot 10^{-3}$ | $0.15 \cdot 10^{-3}$ | $0.30 \cdot 10^{-3}$ | $0.45 \cdot 10^{-3}$ | $0.26 \cdot 10^{-5}$ | $0.33 \cdot 10^{-5}$ | $0.35 \cdot 10^{-5}$ | $0.35 \cdot 10^{-5}$ |
| | 1 h | $0.28 \cdot 10^{-3}$ | $0.31 \cdot 10^{-3}$ | $0.56 \cdot 10^{-3}$ | $0.80 \cdot 10^{-3}$ | $0.50 \cdot 10^{-5}$ | $0.65 \cdot 10^{-5}$ | $0.66 \cdot 10^{-5}$ | $0.67 \cdot 10^{-5}$ |
| | 2 h | $0.64 \cdot 10^{-3}$ | $0.69 \cdot 10^{-3}$ | $0.11 \cdot 10^{-2}$ | $0.15 \cdot 10^{-2}$ | $0.86 \cdot 10^{-5}$ | $0.11 \cdot 10^{-4}$ | $0.12 \cdot 10^{-4}$ | $0.12 \cdot 10^{-4}$ |
| | 6 h | $0.21 \cdot 10^{-2}$ | $0.22 \cdot 10^{-2}$ | $0.33 \cdot 10^{-2}$ | $0.43 \cdot 10^{-2}$ | $0.22 \cdot 10^{-4}$ | $0.30 \cdot 10^{-4}$ | $0.30 \cdot 10^{-4}$ | $0.30 \cdot 10^{-4}$ |
| | 12 h | $0.46 \cdot 10^{-3}$ | $0.49 \cdot 10^{-3}$ | $0.77 \cdot 10^{-3}$ | $0.10 \cdot 10^{-2}$ | $0.36 \cdot 10^{-4}$ | $0.47 \cdot 10^{-4}$ | $0.47 \cdot 10^{-4}$ | $0.47 \cdot 10^{-4}$ |
| | 24 h | $0.12 \cdot 10^{-2}$ | $0.13 \cdot 10^{-2}$ | $0.20 \cdot 10^{-2}$ | $0.25 \cdot 10^{-2}$ | $0.68 \cdot 10^{-4}$ | $0.85 \cdot 10^{-4}$ | $0.87 \cdot 10^{-4}$ | $0.88 \cdot 10^{-4}$ |

Table 4: Root Mean Square Errors for the GARCH (G) and the last period historical volatility (HV) for the 1, 2, 12 and 24 steps ahead predictions.

| rate | time int. | G(1) | G(2) | G(12) | G(24) | HV(1) | HV(2) | HV(12) | HV(24) |
|---|---|---|---|---|---|---|---|---|---|
| CHF/USD | 10 m | $0.61 \cdot 10^{-4}$ | $0.69 \cdot 10^{-4}$ | $0.14 \cdot 10^{-3}$ | $0.21 \cdot 10^{-3}$ | $0.18 \cdot 10^{-5}$ | $0.24 \cdot 10^{-5}$ | $0.25 \cdot 10^{-5}$ | $0.25 \cdot 10^{-5}$ |
| | 20 m | $0.15 \cdot 10^{-3}$ | $0.16 \cdot 10^{-3}$ | $0.33 \cdot 10^{-3}$ | $0.47 \cdot 10^{-3}$ | $0.27 \cdot 10^{-5}$ | $0.36 \cdot 10^{-5}$ | $0.37 \cdot 10^{-5}$ | $0.37 \cdot 10^{-5}$ |
| | 30 m | $0.28 \cdot 10^{-3}$ | $0.31 \cdot 10^{-3}$ | $0.62 \cdot 10^{-3}$ | $0.84 \cdot 10^{-3}$ | $0.40 \cdot 10^{-5}$ | $0.54 \cdot 10^{-5}$ | $0.55 \cdot 10^{-5}$ | $0.55 \cdot 10^{-5}$ |
| | 1 h | $0.58 \cdot 10^{-3}$ | $0.63 \cdot 10^{-3}$ | $0.10 \cdot 10^{-2}$ | $0.13 \cdot 10^{-2}$ | $0.74 \cdot 10^{-5}$ | $0.97 \cdot 10^{-5}$ | $0.99 \cdot 10^{-5}$ | $0.99 \cdot 10^{-5}$ |
| | 2 h | $0.11 \cdot 10^{-2}$ | $0.12 \cdot 10^{-2}$ | $0.17 \cdot 10^{-2}$ | $0.22 \cdot 10^{-2}$ | $0.15 \cdot 10^{-4}$ | $0.19 \cdot 10^{-4}$ | $0.19 \cdot 10^{-4}$ | $0.19 \cdot 10^{-4}$ |
| | 6 h | $0.39 \cdot 10^{-3}$ | $0.41 \cdot 10^{-3}$ | $0.58 \cdot 10^{-3}$ | $0.73 \cdot 10^{-3}$ | $0.34 \cdot 10^{-4}$ | $0.43 \cdot 10^{-4}$ | $0.44 \cdot 10^{-4}$ | $0.44 \cdot 10^{-4}$ |
| | 12 h | $0.74 \cdot 10^{-3}$ | $0.77 \cdot 10^{-3}$ | $0.11 \cdot 10^{-2}$ | $0.14 \cdot 10^{-2}$ | $0.55 \cdot 10^{-4}$ | $0.70 \cdot 10^{-4}$ | $0.72 \cdot 10^{-4}$ | $0.72 \cdot 10^{-4}$ |
| | 24 h | $0.18 \cdot 10^{-3}$ | $0.18 \cdot 10^{-3}$ | $0.24 \cdot 10^{-3}$ | $0.29 \cdot 10^{-3}$ | $0.10 \cdot 10^{-3}$ | $0.14 \cdot 10^{-3}$ | $0.14 \cdot 10^{-3}$ | $0.14 \cdot 10^{-3}$ |
| FRF/USD | 10 m | $0.34 \cdot 10^{-4}$ | $0.39 \cdot 10^{-4}$ | $0.89 \cdot 10^{-4}$ | $0.14 \cdot 10^{-3}$ | $0.12 \cdot 10^{-5}$ | $0.17 \cdot 10^{-5}$ | $0.17 \cdot 10^{-5}$ | $0.17 \cdot 10^{-5}$ |
| | 20 m | $0.90 \cdot 10^{-4}$ | $0.10 \cdot 10^{-3}$ | $0.23 \cdot 10^{-3}$ | $0.36 \cdot 10^{-3}$ | $0.33 \cdot 10^{-5}$ | $0.44 \cdot 10^{-5}$ | $0.44 \cdot 10^{-5}$ | $0.44 \cdot 10^{-5}$ |
| | 30 m | $0.17 \cdot 10^{-3}$ | $0.20 \cdot 10^{-3}$ | $0.43 \cdot 10^{-3}$ | $0.63 \cdot 10^{-3}$ | $0.45 \cdot 10^{-5}$ | $0.60 \cdot 10^{-5}$ | $0.61 \cdot 10^{-5}$ | $0.61 \cdot 10^{-5}$ |
| | 1 h | $0.37 \cdot 10^{-3}$ | $0.40 \cdot 10^{-3}$ | $0.68 \cdot 10^{-3}$ | $0.93 \cdot 10^{-3}$ | $0.71 \cdot 10^{-5}$ | $0.96 \cdot 10^{-5}$ | $0.97 \cdot 10^{-5}$ | $0.97 \cdot 10^{-5}$ |
| | 2 h | $0.61 \cdot 10^{-3}$ | $0.64 \cdot 10^{-3}$ | $0.91 \cdot 10^{-3}$ | $0.12 \cdot 10^{-2}$ | $0.13 \cdot 10^{-4}$ | $0.17 \cdot 10^{-4}$ | $0.17 \cdot 10^{-4}$ | $0.17 \cdot 10^{-4}$ |
| | 6 h | $0.18 \cdot 10^{-2}$ | $0.19 \cdot 10^{-2}$ | $0.26 \cdot 10^{-2}$ | $0.34 \cdot 10^{-2}$ | $0.31 \cdot 10^{-4}$ | $0.40 \cdot 10^{-4}$ | $0.40 \cdot 10^{-4}$ | $0.41 \cdot 10^{-4}$ |
| | 12 h | $0.51 \cdot 10^{-3}$ | $0.54 \cdot 10^{-3}$ | $0.78 \cdot 10^{-3}$ | $0.10 \cdot 10^{-2}$ | $0.49 \cdot 10^{-4}$ | $0.64 \cdot 10^{-4}$ | $0.64 \cdot 10^{-4}$ | $0.64 \cdot 10^{-4}$ |
| | 24 h | $0.10 \cdot 10^{-2}$ | $0.11 \cdot 10^{-2}$ | $0.17 \cdot 10^{-2}$ | $0.22 \cdot 10^{-2}$ | $0.80 \cdot 10^{-4}$ | $0.10 \cdot 10^{-3}$ | $0.10 \cdot 10^{-3}$ | $0.10 \cdot 10^{-3}$ |

Table 5: Root Mean Square Errors for the GARCH (G) and the last period historical volatility (HV) for the 1, 2, 12 and 24 steps ahead predictions.

| rate | time int. | $b_0$ (G) | $b_1$ (G) | $R^2$ (G) | $b_0$ (HV) | $b_1$ (HV) | $R^2$ (HV) |
|---|---|---|---|---|---|---|---|
| USD/DEM | 10 m | $-0.93 \cdot 10^{-5}$ | 0.225 | 0.004 | $0.31 \cdot 10^{-6}$ | 0.049 | 0.002 |
| | | $(0.29 \cdot 10^{-5})$ | (0.069) | | $(0.15 \cdot 10^{-7})$ | (0.040) | |
| | 20 m | $-0.35 \cdot 10^{-4}$ | 0.302 | 0.008 | $0.57 \cdot 10^{-6}$ | 0.100 | 0.010 |
| | | $(0.12 \cdot 10^{-4})$ | (0.104) | | $(0.51 \cdot 10^{-7})$ | (0.080) | |
| | 30 m | $-0.64 \cdot 10^{-4}$ | 0.290 | 0.006 | $0.88 \cdot 10^{-6}$ | 0.063 | 0.004 |
| | | $(0.17 \cdot 10^{-4})$ | (0.077) | | $(0.45 \cdot 10^{-7})$ | (0.044) | |
| | 1 h | $-0.17 \cdot 10^{-1}$ | 0.485 | 0.009 | $0.17 \cdot 10^{-5}$ | 0.057 | 0.003 |
| | | $(0.22 \cdot 10^{-4})$ | (0.060) | | $(0.78 \cdot 10^{-7})$ | (0.031) | |
| | 2 h | $-0.40 \cdot 10^{-3}$ | 0.586 | 0.013 | $0.34 \cdot 10^{-5}$ | 0.050 | 0.003 |
| | | $(0.48 \cdot 10^{-4})$ | (0.070) | | $(0.14 \cdot 10^{-6})$ | (0.016) | |
| | 6 h | $-0.19 \cdot 10^{-2}$ | 0.803 | 0.025 | $0.10 \cdot 10^{-4}$ | 0.037 | 0.001 |
| | | $(0.30 \cdot 10^{-3})$ | (0.124) | | $(0.46 \cdot 10^{-6})$ | (0.014) | |
| | 12 h | $-0.52 \cdot 10^{-3}$ | 0.894 | 0.035 | $0.19 \cdot 10^{-4}$ | 0.093 | 0.009 |
| | | $(0.91 \cdot 10^{-4})$ | (0.151) | | $(0.11 \cdot 10^{-5})$ | (0.034) | |
| | 24 h | $-0.85 \cdot 10^{-3}$ | 0.741 | 0.023 | $0.40 \cdot 10^{-4}$ | 0.100 | 0.010 |
| | | $(0.22 \cdot 10^{-3})$ | (0.182) | | $(0.28 \cdot 10^{-5})$ | (0.052) | |
| USD/GBP | 10 m | $-0.35 \cdot 10^{-4}$ | 0.630 | 0.058 | $0.24 \cdot 10^{-6}$ | 0.173 | 0.030 |
| | | $(0.41 \cdot 10^{-5})$ | (0.073) | | $(0.16 \cdot 10^{-7})$ | (0.057) | |
| | 20 m | $-0.77 \cdot 10^{-4}$ | 0.539 | 0.031 | $0.51 \cdot 10^{-6}$ | 0.145 | 0.021 |
| | | $(0.78 \cdot 10^{-5})$ | (0.054) | | $(0.26 \cdot 10^{-7})$ | (0.042) | |
| | 30 m | $-0.16 \cdot 10^{-3}$ | 0.624 | 0.039 | $0.76 \cdot 10^{-6}$ | 0.162 | 0.026 |
| | | $(0.17 \cdot 10^{-4})$ | (0.066) | | $(0.24 \cdot 10^{-7})$ | (0.026) | |
| | 1 h | $-0.39 \cdot 10^{-3}$ | 0.840 | 0.036 | $0.16 \cdot 10^{-5}$ | 0.103 | 0.011 |
| | | $(0.47 \cdot 10^{-4})$ | (0.099) | | $(0.71 \cdot 10^{-7})$ | (0.035) | |
| | 2 h | $-0.87 \cdot 10^{-3}$ | 0.937 | 0.042 | $0.30 \cdot 10^{-5}$ | 0.147 | 0.022 |
| | | $(0.13 \cdot 10^{-3})$ | (0.144) | | $(0.16 \cdot 10^{-6})$ | (0.044) | |
| | 6 h | $-0.29 \cdot 10^{-2}$ | 0.010 | 0.044 | $0.93 \cdot 10^{-5}$ | 0.137 | 0.019 |
| | | $(0.52 \cdot 10^{-3})$ | (0.181) | | $(0.58 \cdot 10^{-6})$ | (0.050) | |
| | 12 h | $-0.77 \cdot 10^{-3}$ | 0.111 | 0.049 | $0.18 \cdot 10^{-4}$ | 0.195 | 0.038 |
| | | $(0.17 \cdot 10^{-3})$ | (0.243) | | $(0.21 \cdot 10^{-5})$ | (0.092) | |
| | 24 h | $-0.13 \cdot 10^{-2}$ | 0.884 | 0.026 | $0.40 \cdot 10^{-4}$ | 0.159 | 0.025 |
| | | $(0.48 \cdot 10^{-3})$ | (0.306) | | $(0.39 \cdot 10^{-5})$ | (0.025) | |
| JPY/USD | 10 m | $-0.23 \cdot 10^{-4}$ | 0.579 | 0.046 | $0.21 \cdot 10^{-6}$ | 0.184 | 0.034 |
| | | $(0.14 \cdot 10^{-5})$ | (0.036) | | $(0.87 \cdot 10^{-8})$ | (0.035) | |
| | 20 m | $-0.48 \cdot 10^{-4}$ | 0.602 | 0.049 | $0.41 \cdot 10^{-6}$ | 0.167 | 0.028 |
| | | $(0.38 \cdot 10^{-5})$ | (0.047) | | $(0.17 \cdot 10^{-7})$ | (0.034) | |
| | 30 m | $-0.81 \cdot 10^{-4}$ | 0.603 | 0.041 | $0.60 \cdot 10^{-6}$ | 0.148 | 0.022 |
| | | $(0.44 \cdot 10^{-5})$ | (0.033) | | $(0.13 \cdot 10^{-7})$ | (0.017) | |
| | 1 h | $-0.18 \cdot 10^{-3}$ | 0.638 | 0.035 | $0.12 \cdot 10^{-5}$ | 0.117 | 0.014 |
| | | $(0.19 \cdot 10^{-4})$ | (0.067) | | $(0.42 \cdot 10^{-7})$ | (0.027) | |
| | 2 h | $-0.45 \cdot 10^{-3}$ | 0.711 | 0.035 | $0.23 \cdot 10^{-5}$ | 0.121 | 0.015 |
| | | $(0.51 \cdot 10^{-4})$ | (0.080) | | $(0.90 \cdot 10^{-7})$ | (0.027) | |
| | 6 h | $-0.17 \cdot 10^{-2}$ | 0.805 | 0.033 | $0.61 \cdot 10^{-5}$ | 0.226 | 0.051 |
| | | $(0.30 \cdot 10^{-3})$ | (0.143) | | $(0.62 \cdot 10^{-6})$ | (0.086) | |
| | 12 h | $-0.37 \cdot 10^{-3}$ | 0.815 | 0.039 | $0.13 \cdot 10^{-4}$ | 0.125 | 0.016 |
| | | $(0.48 \cdot 10^{-4})$ | (0.101) | | $(0.77 \cdot 10^{-6})$ | (0.032) | |
| | 24 h | $-0.97 \cdot 10^{-3}$ | 0.814 | 0.030 | $0.28 \cdot 10^{-4}$ | 0.061 | 0.004 |
| | | $(0.18 \cdot 10^{-3})$ | (0.144) | | $(0.20 \cdot 10^{-5})$ | (0.031) | |

Table 6: Efficiency test for the GARCH (G) and the last period historical volatility (HV). Corrected standard errors are in parenthesis.

| rate | time int. | $b_0$ (G) | $b_1$ (G) | $R^2$ (G) | $b_0$ (HV) | $b_1$ (HV) | $R^2$ (HV) |
|---|---|---|---|---|---|---|---|
| CHF/USD | 10 m | $-0.29 \cdot 10^{-4}$ | 0.484 | 0.024 | $0.32 \cdot 10^{-6}$ | 0.144 | 0.021 |
| | | $(0.29 \cdot 10^{-5})$ | (0.048) | | $(0.14 \cdot 10^{-7})$ | (0.039) | |
| | 20 m | $-0.90 \cdot 10^{-4}$ | 0.621 | 0.042 | $0.59 \cdot 10^{-6}$ | 0.200 | 0.040 |
| | | $(0.87 \cdot 10^{-5})$ | (0.059) | | $(0.22 \cdot 10^{-7})$ | (0.031) | |
| | 30 m | $-0.15 \cdot 10^{-3}$ | 0.530 | 0.026 | $0.96 \cdot 10^{-6}$ | 0.126 | 0.016 |
| | | $(0.17 \cdot 10^{-4})$ | (0.061) | | $(0.43 \cdot 10^{-7})$ | (0.040) | |
| | 1 h | $-0.39 \cdot 10^{-3}$ | 0.662 | 0.022 | $0.19 \cdot 10^{-5}$ | 0.105 | 0.011 |
| | | $(0.37 \cdot 10^{-4})$ | (0.064) | | $(0.66 \cdot 10^{-7})$ | (0.032) | |
| | 2 h | $-0.78 \cdot 10^{-3}$ | 0.692 | 0.018 | $0.39 \cdot 10^{-5}$ | 0.065 | 0.004 |
| | | $(0.93 \cdot 10^{-4})$ | (0.082) | | $(0.12 \cdot 10^{-6})$ | (0.019) | |
| | 6 h | $-0.33 \cdot 10^{-3}$ | 0.852 | 0.026 | $0.12 \cdot 10^{-4}$ | 0.066 | 0.004 |
| | | $(0.52 \cdot 10^{-4})$ | (0.130) | | $(0.49 \cdot 10^{-6})$ | (0.024) | |
| | 12 h | $-0.60 \cdot 10^{-3}$ | 0.824 | 0.024 | $0.24 \cdot 10^{-4}$ | 0.069 | 0.005 |
| | | $(0.12 \cdot 10^{-3})$ | (0.161) | | $(0.11 \cdot 10^{-5})$ | (0.032) | |
| | 24 h | $-0.91 \cdot 10^{-4}$ | 0.712 | 0.014 | $0.41 \cdot 10^{-4}$ | 0.174 | 0.030 |
| | | $(0.46 \cdot 10^{-4})$ | (0.241) | | $(0.32 \cdot 10^{-5})$ | (0.072) | |
| FRF/USD | 10 m | $-0.18 \cdot 10^{-4}$ | 0.530 | 0.041 | $0.19 \cdot 10^{-6}$ | 0.248 | 0.062 |
| | | $(0.51 \cdot 10^{-5})$ | (0.149) | | $(0.30 \cdot 10^{-7})$ | (0.121) | |
| | 20 m | $-0.24 \cdot 10^{-4}$ | 0.268 | 0.007 | $0.50 \cdot 10^{-6}$ | 0.058 | 0.003 |
| | | $(0.56 \cdot 10^{-5})$ | (0.062) | | $(0.22 \cdot 10^{-7})$ | (0.038) | |
| | 30 m | $-0.58 \cdot 10^{-4}$ | 0.346 | 0.013 | $0.74 \cdot 10^{-6}$ | 0.094 | 0.009 |
| | | $(0.13 \cdot 10^{-4})$ | (0.078) | | $(0.48 \cdot 10^{-7})$ | (0.058) | |
| | 1 h | $-0.18 \cdot 10^{-3}$ | 0.489 | 0.013 | $0.15 \cdot 10^{-5}$ | 0.096 | 0.009 |
| | | $(0.22 \cdot 10^{-4})$ | (0.061) | | $(0.83 \cdot 10^{-7})$ | (0.049) | |
| | 2 h | $-0.40 \cdot 10^{-3}$ | 0.666 | 0.019 | $0.29 \cdot 10^{-5}$ | 0.109 | 0.012 |
| | | $(0.38 \cdot 10^{-4})$ | (0.061) | | $(0.15 \cdot 10^{-6})$ | (0.043) | |
| | 6 h | $-0.14 \cdot 10^{-2}$ | 0.762 | 0.025 | $0.94 \cdot 10^{-5}$ | 0.060 | 0.004 |
| | | $(0.22 \cdot 10^{-3})$ | (0.119) | | $(0.41 \cdot 10^{-6})$ | (0.018) | |
| | 12 h | $-0.45 \cdot 10^{-3}$ | 0.897 | 0.041 | $0.18 \cdot 10^{-4}$ | 0.106 | 0.011 |
| | | $(0.76 \cdot 10^{-4})$ | (0.146) | | $(0.97 \cdot 10^{-6})$ | (0.036) | |
| | 24 h | $-0.87 \cdot 10^{-3}$ | 0.853 | 0.042 | $0.36 \cdot 10^{-4}$ | 0.079 | 0.006 |
| | | $(0.18 \cdot 10^{-3})$ | (0.167) | | $(0.22 \cdot 10^{-5})$ | (0.029) | |

Table 7: Efficiency test for the GARCH (G) and the last period historical volatility (HV). Corrected standard errors are in parenthesis.

| year | semester | G(1) | G(2) | G(12) | G(24) | HV(1) | HV(2) | HV(12) | HV(24) |
|---|---|---|---|---|---|---|---|---|---|
| 1987 | 1st | $0.26 \cdot 10^{-3}$ | $0.27 \cdot 10^{-3}$ | $0.42 \cdot 10^{-3}$ | $0.60 \cdot 10^{-3}$ | $0.44 \cdot 10^{-5}$ | $0.57 \cdot 10^{-5}$ | $0.58 \cdot 10^{-5}$ | $0.60 \cdot 10^{-5}$ |
| | 2nd | $0.59 \cdot 10^{-3}$ | $0.60 \cdot 10^{-3}$ | $0.70 \cdot 10^{-3}$ | $0.81 \cdot 10^{-3}$ | $0.12 \cdot 10^{-4}$ | $0.16 \cdot 10^{-4}$ | $0.17 \cdot 10^{-4}$ | $0.16 \cdot 10^{-4}$ |
| 1988 | 1st | $0.55 \cdot 10^{-3}$ | $0.62 \cdot 10^{-3}$ | $0.13 \cdot 10^{-2}$ | $0.18 \cdot 10^{-2}$ | $0.74 \cdot 10^{-5}$ | $0.99 \cdot 10^{-5}$ | $0.99 \cdot 10^{-5}$ | $0.10 \cdot 10^{-4}$ |
| | 2nd | $0.79 \cdot 10^{-3}$ | $0.86 \cdot 10^{-3}$ | $0.14 \cdot 10^{-2}$ | $0.18 \cdot 10^{-2}$ | $0.58 \cdot 10^{-5}$ | $0.80 \cdot 10^{-5}$ | $0.79 \cdot 10^{-5}$ | $0.82 \cdot 10^{-5}$ |
| 1989 | 1st | $0.38 \cdot 10^{-3}$ | $0.39 \cdot 10^{-3}$ | $0.50 \cdot 10^{-3}$ | $0.61 \cdot 10^{-3}$ | $0.11 \cdot 10^{-4}$ | $0.14 \cdot 10^{-4}$ | $0.15 \cdot 10^{-4}$ | $0.15 \cdot 10^{-4}$ |
| | 2nd | $0.20 \cdot 10^{-2}$ | $0.23 \cdot 10^{-2}$ | $0.29 \cdot 10^{-2}$ | $0.29 \cdot 10^{-2}$ | $0.86 \cdot 10^{-5}$ | $0.11 \cdot 10^{-4}$ | $0.12 \cdot 10^{-4}$ | $0.12 \cdot 10^{-4}$ |
| 1990 | 1st | $0.60 \cdot 10^{-3}$ | $0.62 \cdot 10^{-3}$ | $0.80 \cdot 10^{-3}$ | $0.99 \cdot 10^{-3}$ | $0.38 \cdot 10^{-5}$ | $0.50 \cdot 10^{-5}$ | $0.46 \cdot 10^{-5}$ | $0.50 \cdot 10^{-5}$ |
| | 2nd | $0.19 \cdot 10^{-2}$ | $0.23 \cdot 10^{-2}$ | $0.26 \cdot 10^{-2}$ | $0.26 \cdot 10^{-2}$ | $0.57 \cdot 10^{-5}$ | $0.71 \cdot 10^{-5}$ | $0.74 \cdot 10^{-5}$ | $0.79 \cdot 10^{-5}$ |
| 1991 | 1st | $0.80 \cdot 10^{-3}$ | $0.83 \cdot 10^{-3}$ | $0.11 \cdot 10^{-2}$ | $0.14 \cdot 10^{-2}$ | $0.13 \cdot 10^{-4}$ | $0.16 \cdot 10^{-4}$ | $0.17 \cdot 10^{-4}$ | $0.17 \cdot 10^{-4}$ |
| | 2nd | $0.34 \cdot 10^{-2}$ | $0.37 \cdot 10^{-2}$ | $0.38 \cdot 10^{-2}$ | $0.38 \cdot 10^{-2}$ | $0.89 \cdot 10^{-5}$ | $0.13 \cdot 10^{-4}$ | $0.13 \cdot 10^{-4}$ | $0.13 \cdot 10^{-4}$ |
| 1992 | 1st | $0.50 \cdot 10^{-3}$ | $0.51 \cdot 10^{-3}$ | $0.60 \cdot 10^{-3}$ | $0.70 \cdot 10^{-3}$ | $0.46 \cdot 10^{-5}$ | $0.60 \cdot 10^{-5}$ | $0.61 \cdot 10^{-5}$ | $0.62 \cdot 10^{-5}$ |
| | 2nd | $0.82 \cdot 10^{-3}$ | $0.87 \cdot 10^{-3}$ | $0.13 \cdot 10^{-2}$ | $0.19 \cdot 10^{-2}$ | $0.15 \cdot 10^{-4}$ | $0.19 \cdot 10^{-4}$ | $0.18 \cdot 10^{-4}$ | $0.18 \cdot 10^{-4}$ |
| 1993 | 1st | $0.23 \cdot 10^{-2}$ | $0.26 \cdot 10^{-2}$ | $0.26 \cdot 10^{-2}$ | $0.26 \cdot 10^{-2}$ | $0.72 \cdot 10^{-5}$ | $0.95 \cdot 10^{-5}$ | $0.95 \cdot 10^{-5}$ | $0.95 \cdot 10^{-5}$ |
| | 2nd | $0.13 \cdot 10^{-2}$ | $0.15 \cdot 10^{-2}$ | $0.21 \cdot 10^{-2}$ | $0.21 \cdot 10^{-2}$ | $0.63 \cdot 10^{-5}$ | $0.82 \cdot 10^{-5}$ | $0.85 \cdot 10^{-5}$ | $0.83 \cdot 10^{-5}$ |

Table 8: Root Mean Square Errors for the GARCH (G) and the last period historical volatility (HV) for the 1, 2, 12 and 24 steps ahead predictions. The currency is the USD/DEM at the 2 hours interval.

| year | semester | $b_0$ (G) | $b_1$ (G) | $R^2$ (G) | $b_0$ (HV) | $b_1$ (HV) | $R^2$(HV) |
|---|---|---|---|---|---|---|---|
| 1987 | 1st | $-0.16 \cdot 10^{-3}$ | 0.638 | 0.028 | $0.17 \cdot 10^{-5}$ | 0.188 | 0.035 |
| | | $0.38 \cdot 10^{-4}$ | 0.147 | | $0.14 \cdot 10^{-6}$ | 0.061 | |
| | 2nd | $-0.40 \cdot 10^{-3}$ | 0.675 | 0.007 | $0.34 \cdot 10^{-5}$ | 0.074 | 0.005 |
| | | $0.15 \cdot 10^{-3}$ | 0.252 | | $0.38 \cdot 10^{-6}$ | 0.053 | |
| 1988 | 1st | $-0.72 \cdot 10^{-4}$ | 0.134 | 0.001 | $0.13 \cdot 10^{-5}$ | 0.027 | 0.001 |
| | | $0.34 \cdot 10^{-4}$ | 0.063 | | $0.22 \cdot 10^{-6}$ | 0.017 | |
| | 2nd | $-0.46 \cdot 10^{-3}$ | 0.576 | 0.015 | $0.17 \cdot 10^{-5}$ | 0.195 | 0.038 |
| | | $0.19 \cdot 10^{-3}$ | 0.242 | | $0.23 \cdot 10^{-6}$ | 0.094 | |
| 1989 | 1st | $-0.32 \cdot 10^{-3}$ | 0.831 | 0.028 | $0.33 \cdot 10^{-5}$ | 0.034 | 0.001 |
| | | $0.57 \cdot 10^{-4}$ | 0.149 | | $0.33 \cdot 10^{-6}$ | 0.022 | |
| | 2nd | $-0.20 \cdot 10^{-2}$ | 0.996 | 0.037 | $0.22 \cdot 10^{-5}$ | 0.174 | 0.030 |
| | | $0.71 \cdot 10^{-3}$ | 0.354 | | $0.21 \cdot 10^{-6}$ | 0.091 | |
| 1990 | 1st | $-0.33 \cdot 10^{-3}$ | 0.558 | 0.006 | $0.13 \cdot 10^{-5}$ | 0.054 | 0.003 |
| | | $0.14 \cdot 10^{-3}$ | 0.235 | | $0.99 \cdot 10^{-7}$ | 0.029 | |
| | 2nd | $-0.75 \cdot 10^{-3}$ | 0.397 | 0.007 | $0.23 \cdot 10^{-5}$ | 0.057 | 0.003 |
| | | $0.31 \cdot 10^{-3}$ | 0.163 | | $0.17 \cdot 10^{-6}$ | 0.030 | |
| 1991 | 1st | $-0.46 \cdot 10^{-3}$ | 0.581 | 0.013 | $0.39 \cdot 10^{-5}$ | 0.093 | 0.009 |
| | | $0.12 \cdot 10^{-3}$ | 0.148 | | $0.35 \cdot 10^{-6}$ | 0.026 | |
| | 2nd | $-0.89 \cdot 10^{-2}$ | 0.607 | 0.056 | $0.22 \cdot 10^{-5}$ | 0.239 | 0.057 |
| | | $0.68 \cdot 10^{-2}$ | 0.003 | | $0.41 \cdot 10^{-6}$ | 0.183 | |
| 1992 | 1st | $-0.13 \cdot 10^{-3}$ | 0.271 | 0.001 | $0.18 \cdot 10^{-5}$ | 0.024 | 0.001 |
| | | $0.18 \cdot 10^{-3}$ | 0.352 | | $0.15 \cdot 10^{-6}$ | 0.032 | |
| | 2nd | $-0.53 \cdot 10^{-3}$ | 0.656 | 0.030 | $0.52 \cdot 10^{-5}$ | 0.051 | 0.003 |
| | | $0.93 \cdot 10^{-4}$ | 0.113 | | $0.44 \cdot 10^{-6}$ | 0.034 | |
| 1993 | 1st | $-0.62 \cdot 10^{-4}$ | 0.028 | 0.000 | $0.24 \cdot 10^{-5}$ | 0.003 | 0.000 |
| | | $0.34 \cdot 10^{-3}$ | 0.147 | | $0.22 \cdot 10^{-6}$ | 0.017 | |
| | 2nd | $-0.54 \cdot 10^{-3}$ | 0.418 | 0.009 | $0.19 \cdot 10^{-5}$ | 0.038 | 0.001 |
| | | $0.32 \cdot 10^{-3}$ | 0.252 | | $0.20 \cdot 10^{-6}$ | 0.040 | |

Table 9: Efficiency test for the GARCH (G) and the last period historical volatility (HV). Corrected standard errors are in parenthesis. The currency is the USD/DEM at the 2 hours interval.