Unveiling Non Linearities Through Time Scale Transformations

Dominique M. Guillaume, Olivier V. Pictet,
Ulrich A. Müller, Michel M. Dacorogna

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Contents

1 Introduction **1**

2 Methodology **1**

2.1 Diagnostic tools 1

2.2 Time scale transformations 2

3 Results **4**

3.1 The correlogram study 4

3.2 The BDS-test results 5

4 Conclusion **8**

REFERENCES **9**

Abstract

In this paper, we show that intra-daily foreign exchange rate returns exhibit even stronger nonlinearities than daily or weekly returns. These nonlinearities result from the intra-daily seasonality and the presence of market participants with different time-horizons. Moreover, we present some evidence that both these nonlinearities and the autocorrelation patterns of the volatility can successfully be accounted for by successive time scale transformations.

1 Introduction

The work of Mandelbrot and Taylor (1967), Clark (1972) and Allais (1973) have introduced the concept of time deformation to model the subjective view of time held by market participants. In these models, time speeds up when the information flow rises. This information flow can be measured by the interest rate (Allais), the transaction volume (Mandelbrot and Taylor) or the volatility (Clark). These new time scales were shown to fit quite adequately the properties of the time series. Rather than the physical time, they are thus the "natural" or relevant time scale in which to measure the price generating process. More recently, Stock (1988) showed that in such time scale prices could be linearly related to the underlying fundamentals. Stock also demonstrated that this time scale could capture the conditional heteroskedasticity exhibited by the time series. With the changing volume of transactions at the intra-daily frequencies, the use of such time scales seems even more relevant (Dacorogna et al., 1993b; Ghysels and Jasiak, 1994). Time scale transformations are thus an interesting alternative to the Auto Regressive Conditional Heteroskedastic class of models (Engle, 1982).

In this paper, we want to evaluate the capacity of time scale transformations to capture the very rich structure of intra-daily FX rates (see the survey of Guillaume et al. (1994). In a first step, we use various diagnostic tools to describe this rich structure of FX rates. We also test whether this finer structure at the intra-daily frequencies is indeed reflected in stronger nonlinearities than at the daily and weekly ones (Hsieh, 1989; Hsieh, 1991; Brock et al., 1992). In a second step, once we have identified the various sources of these nonlinearities, we show how most of them can be accounted for by two successive time scale transformations. The first time scale is an intra-day extension of the business time scale implicitly used at daily intervals by omitting week-ends or holidays (Dacorogna et al., 1993b). The second time scale (Dacorogna et al., 1993a) is very similar in spirit to the time deformation models described above.

The remainder of this paper is divided as follows. In the next section, we present our methodology. The third section summarises the results. The fourth section concludes the paper.

2 Methodology

2.1 Diagnostic tools

We use two basic diagnostic tools to describe the nonlinearities present in intra-daily FX rates: correlograms of the returns and the volatility and the test for nonlinearities proposed by Brock, Dechert and Scheinkman (BDS) (1987).

The motivation for the use of *correlograms* comes from the singular value decomposition of intra-daily returns and volatility performed in Guillaume (1995) where the only distinct structure of FX rates appeared to be the auto-correlation present in the volatility. The analysis of correlograms of the volatility given in Dacorogna et al. (1993b) indeed revealed the presence of many features of the FX market.

In addition to correlograms, we use the *BDS* test which has a relatively good power against nonlinearities induced by the auto-correlation of the second moment (Lee et al., 1993). In comparison to other tests of nonlinearities like the bi-spectrum test (Hinich, 1982; Ashley et al., 1986), the BDS test offers the advantage of not depending upon the existence of higher moments. This is an important property since only the three first moments of the distribution of intra-daily FX returns exist as shown in recent studies of the tail indices of these distributions (Dacorogna et al., 1994).

The BDS test is based on the following time-delay reconstruction of the phase space which is topologically equivalent to the original phase space (Takens, 1981):

$$X = \{x(t), x(t+\tau), ..., x(t+\tau(m-1))\} \quad t = 1, ..., n-(m-1)\tau \tag{2.1}$$

where x is the FX return at time t to be defined in section 3 and τ ($\tau \in N$) is a fixed time lag. This lag is usually taken equal to the number of lags corresponding to the first zero of the auto-correlation function to ensure linear independence of the reconstructed variables. Alternatively the time-series could be filtered by an Auto-Regressive process and then take $\tau = 1$.

Define now the correlation integral $C(m, \varepsilon)$ as follows:

$$C(m, \varepsilon) = \frac{1}{n(n-1)} \sum_i \sum_j \underset{i \neq j}{} I(\varepsilon - |X(i) - X(j)|) \tag{2.2}$$

where $I(x)$ is called the Indicator Function and is valued at 1 if the distance between the two points $X(i)$ and $X(j)$ in the m-dimensional space is less than ε, and at 0 if the distance is greater. The BDS statistics is then:

$$\text{BDS} = n^{1/2}[C(m, \varepsilon) - C(1, \varepsilon)^m] \tag{2.3}$$

Under the null hypothesis that the series is independently and identically distributed, the BDS is asymptotically normally distributed with zero mean and a complicated variance whose exact formulation is given in Brock et al. (1987). The parameter ε is usually taken between 0.5 and 1.5 times the standard error of the data[1]. Although the BDS test was originally not meant to distinguish between different types of nonlinearities, we will see in the next section that together with correlograms, it is an interesting diagnostic tool.

2.2 Time scale transformations

In this paper, we use three different time scales. The first one is the physical time used in most empirical studies. In this physical time, data points are usually regularly sampled in order to create an homogeneous time series. Raw intra-daily – tick-by-tick – FX quoted rates, however, arrive at an irregular pace. Therefore, we used linear interpolation over data holes such as in Müller et al. (1990) to obtain regularly spaced data in physical time.

[1] A very fast algorithm for long data series is available from the authors. This algorithm is based on the box counting algorithm of Grassberger et al. (1991) and a quickstep reordering within the boxes.

The second time scale used here is the de-seasonalized Theta (ϑ) time scale introduced by Dacorogna et al. (1993b). This time scale models the intra-daily deterministic seasonal patterns of the volatility caused by the geographical dispersion of market agents. Week-ends, holidays and day-light saving effects are also taken into account by the ϑ-scale as discussed in (Dacorogna et al., 1993b). As shown in Guillaume et al. (1995), modelling these deterministic seasonal patterns through such a time scale transformation eliminates serious mis-specifications of the GARCH process as the conditional heteroskedastic structure of the data becomes more apparent.

Both this ϑ-time scale and the intrinsic time scale introduced below are based on the empirical scaling law relating the average volatility over a time interval to the size of this time interval (Müller et al., 1990):

$$\overline{|r(t_i)|} \;=\; \left(\frac{\Delta t_i}{\Delta T}\right)^{1/\mathrm{E}} \tag{2.4}$$

The drift exponent – 1/E – is a constant which is quite stable over the years and across the currencies (Guillaume et al., 1994a) and ΔT is an empirical time constant depending upon the currency. To compute the ϑ-time scale, the inverse of the scaling law is applied to the hourly average volatility for each hour of the week resulting in the following activity statistics:

$$a_{stat,i} \;\equiv\; \frac{\Delta T}{\Delta t}(\overline{|r(t_i)|})^{\mathrm{E}} \tag{2.5}$$

where $\Delta t = 1$ hour and the index i refers to the hour of the week ($i = 1, ..., 168$). An activity function $a(t)$ is then fitted to the results of the statistics $a_{stat,i}$. This activity function is divided into three components corresponding to the three main geographical FX markets – East-Asia, Europe and America –. Each of these markets is described by an activity variable $a_{0,k}$ corresponding to a constant base level during the closing hours and an activity variable $a_{1,k(t)}$ describing the activity during the opening hours of the corresponding market (k=1, 2, 3). The ϑ-time is then the time integral of the worldwide activity:

$$\vartheta(t) \;\equiv\; a_0\;(t - t_0) + \sum_{k=1}^{3} \int_{t_0}^{t} a_k(t')\;\mathrm{d}t' \tag{2.6}$$

The activity variable is normalized in such a way that ϑ-time can be measured in the same units as physical time (e. g. hours, days, weeks); one full week in ϑ-time corresponds approximately[2] to one week in physical time.

Finally, a third time scale called intrinsic time is used to model the remaining conditional heteroskedasticity. In this intrinsic time, the following activity variable ($a(t_j)$ where t_j refers to the unequally spaced tick-by-tick time intervals) based on the scaling law is used to model the instantaneous volatility instead of the hourly average volatility as in the ϑ-time scale:

$$a(t_i) \;\equiv\; \frac{\Delta T}{\Delta \vartheta_r}(v_r)^{E} \tag{2.7}$$

where v_r is the recent volatility (not annualized) and $\Delta\vartheta_r$ is a range parameter, the ϑ-time interval size of the price changes considered for computing the recent volatility v_r. In this implementation

[2] The actual normalization is done over 4 years to include leap years and to take into account holidays effect in each market.

of intrinsic time, we use a more complex definition of the recent volatility than the one proposed in Dacorogna et al. (1993a), based on a sum of exponential moving averages of the volatility for different recent time horizons (Müller, 1992). The definition also takes care of the problem of holes when data are missing. The intrinsic time is then given by:

$$\Delta\tau(t_j) \equiv \frac{a(t_j)}{\overline{a}_{t_j}} \Delta\vartheta_{t_j} \tag{2.8}$$

where $\Delta\vartheta_{t_j}$ is the corresponding ϑ-time from tick to tick and $\overline{a}_{t_j}$ is the long term mean of $a(t_j)$ computed in a way that intrinsic time flows neither slower nor faster than physical time or ϑ-time in the long-term average. The underlying time scale of the intrinsic time definition is always ϑ-time rather than physical time, thus analyzing the time series in its deseasonalized form. The consequence of using such a scale is to expand periods of high volatility and contract those of low volatility, better capturing the relative importance of events to the market. To capture the temporal heterogeneity of GARCH processes (Guillaume et al., 1995), we used three different time horizons for the range parameter $\Delta\vartheta_r$: 1 hour, 1 day and 1 week. Although intrinsic time incorporates the full information contained in tick-by- tick data, similarly to the ϑ time, it can easily be (temporally) aggregated. Its future flow is, however, not known and can only be obtained by a forecasting model.

3 Results

The basic data for this study are constructed from tick-by-tick data collected in physical time and provided by Reuters from the main FX contributors around the world, 24 hours a day, 7 days a week, for the period from the 01.01.1987 midnight Greenwhich Mean Time (GMT) to the 31.12.1993 midnight (GMT). Outliers are filtered out as in (Dacorogna et al., 1993b; Dacorogna et al., 1994). The variables we use are the returns

$$r(t_i) \equiv [x(t_i) - x(t_i - \Delta t)] \tag{3.1}$$

where t_i indicates a fixed sampling, with the price computed as the average of the logarithm of the bid and ask prices ($x(t_i) = [\log p_{ask}(t_i) + \log p_{bid}(t_i)] / 2$). The volatility is defined as the absolute value of the returns. The FX rates are the main exchange rates against the USD: DEM, JPY, GBP, CHF and FRF.

3.1 The correlogram study

In Figure 2 and Figure 1, several findings derived from the analysis of the correlograms of the returns and the volatility are summarized.

1. An economically negligible but statistically significant first order negative autocorrelation of the returns at the 1 hour interval can be observed both in physical and in ϑ-time. The presence of this negative autocorrelation was first reported by Goodhart (1989) on short time intervals like 1 minute and reflects the oscillation of prices quoted by dealers with diverging opinions on the price or with order imbalances (Dacorogna et al., 1993b; Bollerslev and Domowitz, 1993). As can be seen from Figure 1, the effect of this negative autocorrelation is decreasing as one aggregates data from the 1 minute frequency to lower frequencies and becomes insignificant after 70 minutes.

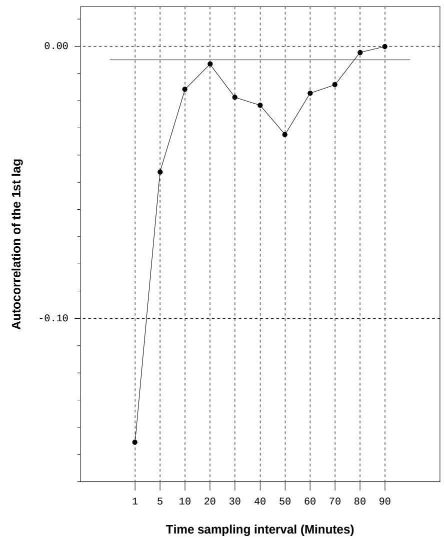

Figure 1: First-order autocorrelation

The sampling period runs from 01.01.87 to 31.12.93. The currency is the DEM/USD. The straight line represents the 95% confidence interval of a Gaussian random process.

2. A positive followed by a negative correlation of the returns is observed at lags corresponding to approximately 1 day in physical time and in ϑ-time. This effect is related to the stochastic seasonality of the volatility at the same lags and corresponds to (stochastic) information spillovers between dealers within the same geographical markets (Guillaume et al., 1994). The stochastic seasonality disappears in intrinsic

The autocorrelation function for the price changes (circle) and the absolute value of the price changes (star) using different time scales: physical time, theta time, intrinsic time on 1 hour, intrinsic time on 1 day and intrinsic time on 1 week. The sampling period runs from 01.01.87 to 31.12.93. The currency is the USD/DEM.

3. The ϑ-time scale eliminates the deterministic seasonality of the volatility present in physical time. It also reveals the long term memory of the volatility which was hidden by the seasonality peaks in the study in physical time.

4. The stochastic seasonality disappears in intrinsic time.

5. Although there is still some (statistically) significant residual autocorrelation, the intrinsic time at 1 hour seems to capture most of the structure contained in the data. This can easily be understood as the most important segment of the market is made of intra-daily short term dealers acting within the 1 hour time horizon. The residual autocorrelation can be explained by the uncertainty on the price resulting from the comparable size of the spread relative to the size of the returns at this frequency (Guillaume et al., 1995).

6. Both the high short term – ARCH type – memory of the volatility and its long term memory are still present in intrinsic time at the 1 day and 1 week intervals, but their effects mostly disappears at lags higher than 2 days and 2 weeks respectively. This results comes from the fact that the behavior of traders at higher frequencies, in particular short term intra-day dealers, is not modelled.

3.2 The BDS-test results

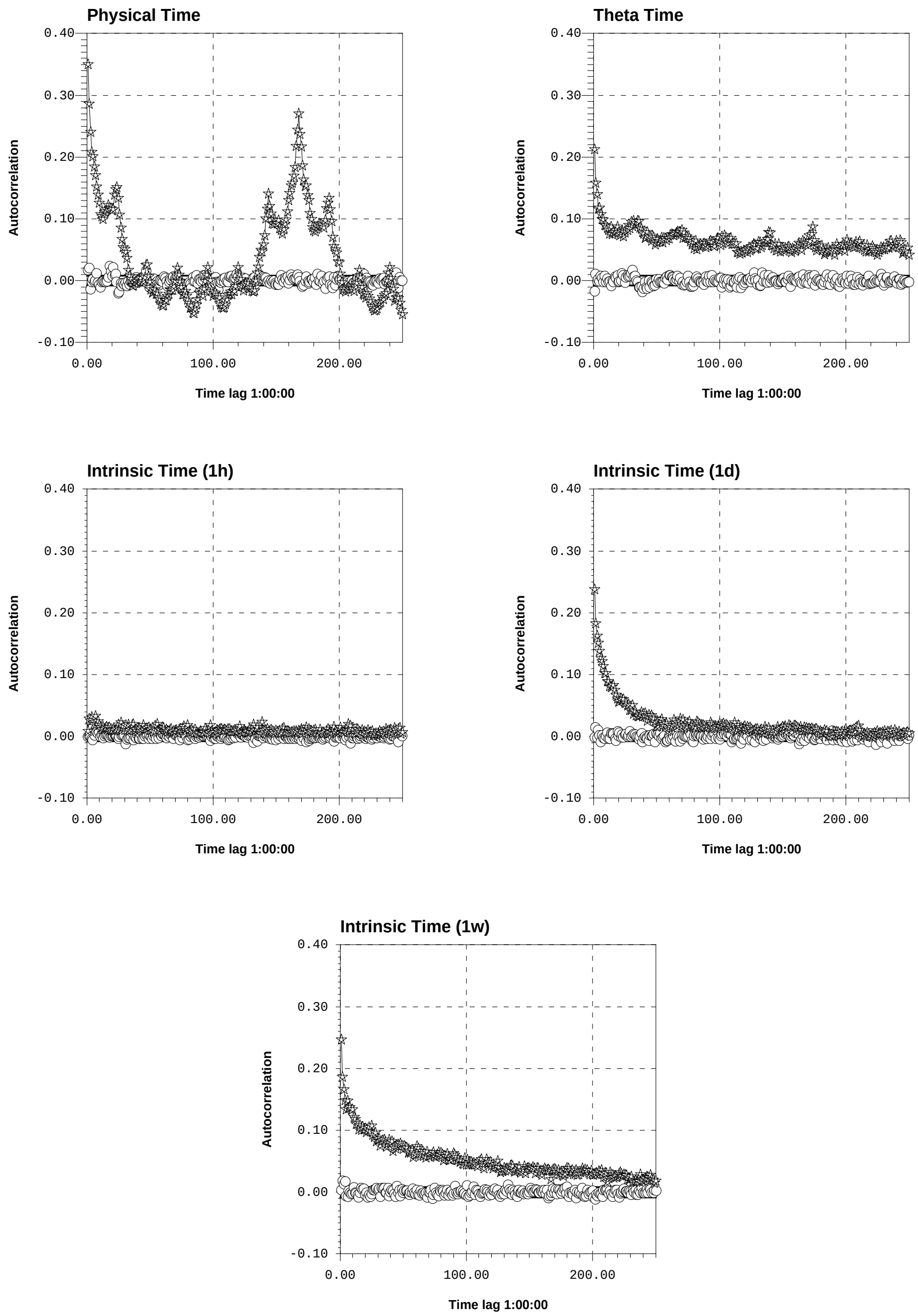

Figure 2: Correlograms

The BDS test is sensitive to the size of the sample under analysis (Brock et al., 1992). In Table 1, the sample size effect on the BDS results for various embedding dimensions is shown for our particular study. A (sub-)sample size of 2500 data points corresponds to the number of daily data contained in the whole 7 years sample. One can observe that this sample size effect is quite important, even though the high level of nonlinearities is not simply the result of this effect. In the last column of Table 1, one can see that the presence of these nonlinearities indeed disappears when the data are re-shuffled by a random permutation. To avoid the sample size

Embedding	Sub-sample Size						
	1250	2500	5000	10000	15000	20000	Shuffled
c(2)	5.4	8.3	12.1	17.4	21.5	25.4	1.0
c(3)	6.3	9.8	14.3	20.6	25.5	31.1	1.4
c(4)	6.9	10.6	15.7	22.6	28.0	34.9	1.4
c(5)	7.4	11.5	17.1	24.4	30.4	38.8	1.4
c(6)	7.9	12.3	18.3	26.2	32.8	42.6	1.3
c(7)	8.4	13.2	19.6	28.1	35.3	46.9	1.2
c(8)	8.9	14.0	20.9	30.0	38.0	51.7	1.0
c(9)	9.5	15.0	22.4	32.1	40.9	57.0	1.1
c(10)	10.1	16.0	24.1	34.6	44.3	63.4	1.0

Table 1: Sample size impact on B.D.S. results

Computation of the mean B.D.S. results for various sub-sample sizes for the USD/DEM at the 2 hours frequency. The sampling period runs from 01.01.87 to 31.12.93. In the last column, the data (sample size of 20,000) were randomly re-shuffled.

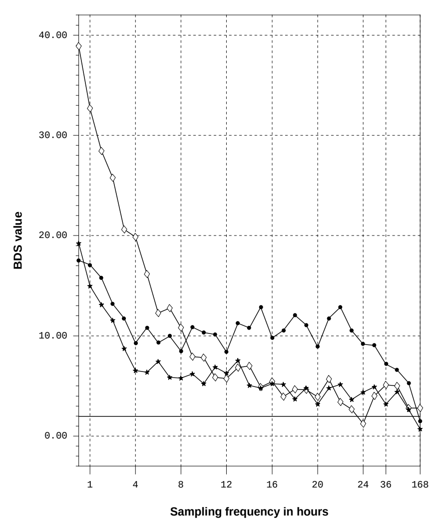

Figure 3: B.D.S. Results for Various Time Scales

B.D.S. results in function of the sampling frequency interval. The embedding dimension is 7 and the value of the radius (ε) is equal to the standard deviation. The sampling period runs from 01.01.87 to 31.12.93. The time scales are physical time (diamond), theta time (circle) and intrinsic time on 1 hour (star).

bias, the results that are shown in Figures 3 and 4 are the mean BDS results for sub-samples of 2500 data points over the whole period.

The results of the BDS test for the different time scales and various sampling frequencies are summarized in Figures 3 and 4. They are clear evidence of the much stronger nonlinearities at the intra-daily frequencies. Moreover, these results corroborate the correlogram analysis made above in section 3.1.

First, the nonlinearities due to market seasonal patterns are clearly reflected by the much higher (lower) value of the BDS in physical (ϑ) time for frequencies up to 8-10 hours, which

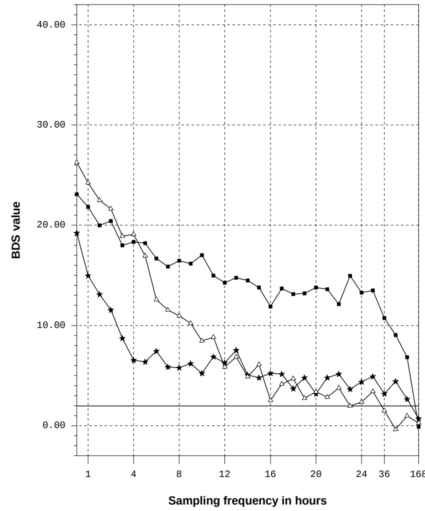

Figure 4: B.D.S. Results for Different Definitions of Intrinsic Time

B.D.S. results in function of the sampling frequency interval. The embedding dimension is 7 and the value of the radius (ε) is equal to the standard deviation. The sampling period runs from 01.01.87 to 31.12.93. The time scales are intrinsic time on 1 hour (star), intrinsic time on 1 day (triangle) and intrinsic time on 1 week (square).

approximately corresponds to the opening period of a market. Second, for frequencies lower than 10 hours, values of the BDS in the ϑ-time scale are larger reflecting the more apparent short term memory of the volatility. Third, values of the BDS in intrinsic time at 1 hour are much lower, but still decreasing for intervals up to 3-4 hours where short-term intra-day dealers are still active. Some residual structure can however still be observed. Fourth, values of the BDS in intrinsic time at 1 day and 1 week are larger up to intervals of 12 hours and 3-4 days respectively reflecting the residual short term memory not modelled by these time scales.

4 Conclusion

In this paper, we have described various sources of nonlinearities present in the intra-daily FX rates: the deterministic and stochastic seasonalities, the short and long term memories, the price formation process taking place at the highest frequencies and the difference in the time-horizons of market participants. We also showed that intra-daily FX rate returns exhibit much stronger nonlinearities than daily or weekly returns.

Furthermore, by introducing two new time scales based on the empirical scaling law that relates the average volatility over a time-interval and this time-interval, we could explain both these nonlinearities and the auto-correlation patterns of the volatility. The first time scale transformation (ϑ-time) removed most of the nonlinearities due to the deterministic seasonality of the different markets. It also made the short and long term memory characteristics of the volatility more apparent. The second time scale transformation (intrinsic time) both modelled the remaining stochastic seasonality and the memory of the volatility corresponding to the time horizon being taken account. In particular, intrinsic time at the 1 hour time horizon could account for most of the structure present in the data except for the very short horizons. This could be easily understood as this time horizon corresponds to short term intra-daily dealers who make up the largest share of the market. However, in addition to the accurate modelling of very short term intra-daily dealers, some important short-comings of these time-scales transformations are the integration of market participants with different time-horizons and their interaction.

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