A Closer Look at the Eurofutures Market: Intraday Statistical Analysis

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Abstract

We investigate the intraday behavior of the prices of Eurofutures contracts traded at the Chicago Mercantile Exchange, the London International Financial Futures Exchange and at the Singapore International Monetary Exchange. Since futures markets are the most liquid for interest rate instruments and they yield high-quality intraday data, it is somehow surprising that their intraday behavior has not been thoroughly studied in the literature.

We compare the behavior of actual trading prices and bid/ask quotes both on an intra-day and intraweek basis.

There are interesting similarities with the foreign exchange market. The value of drift exponent of the scaling law for absolute changes is basically the same for FX rates. As in FX, there are strong daily seasonalities in the volatility. There are also day-of-the week effects.

We find a systematic deterministic volatility pattern depending on the time left before the expiry of the Eurofutures contract. The volatility typically tends to decrease as the expiry becomes closer.

1 Introduction

Interest rate (IR) futures represent one of the best instruments to investigate the properties of interest rates, especially on an intraday basis. Futures markets are very liquid and they yield high-quality intraday data. They are usually characterized by lower frictions and lower transaction costs than cash markets, e.g. the typical spread in the Eurofutures markets is 10-20 times smaller than the quoted spread for Eurodeposits. In addition, the mechanism of price formation for futures is faster than for cash contracts ((Fung and Leung, 1993; Garbade and Silber, 1985)). For all these reasons futures markets supply more accurate and more frequent information on IR than the cash market. The specific object of our study is represented by the price behavior of Eurofutures [1]. A companion paper (Ballocchi et al., 1998) focuses on the multivariate structure of the implied forward rates derived from Eurofutures prices, and provides elements for a model.

Futures contracts are derivatives of an underlying asset. A good definition is given in (Hull, 1993): "A futures contract is an agreement between two parties to buy or sell an asset at a certain time in the future [delivery period] for a certain price." More specifically, "an interest rate futures contract is a futures contract on an asset whose price is dependent solely on the level of interest rates" (Hull, 1993). In case of a Eurofutures contract, the party with the short position does not deliver any asset at expiry; the contract is settled in cash taking as a reference the actual short-term offered rate (LIBOR) on a notional deposit.

Information on IR futures are particularly valuable for financial institutions and above all for banks. A quick analysis of a typical balance sheet would reveal a higher exposure to IR risk than, for example, to foreign exchange (FX) risk. IR futures can also be used as hedging instruments. From a practical point of view, there is a widespread need for a better understanding of the empirical IR futures behavior on an intraday basis; nevertheless, in the current literature there is no published review of the basic facts characterizing the intraday Eurofutures market [2].

The only aspect of intraday analysis that seem to have enjoyed detailed attention is the impact

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[1] Bond futures will be covered in another paper (Ballocchi, 1998) (in preparation).

[2] Studies of limited scope have been performed for instance by P. Lequeux on Singapore International Monetary Exchange (SIMEX) and London International Financial Futures Exchange (LIFFE) data (personal communication).

of scheduled news releases (Ederington and Lee, 1993; Ederington and Lee, 1995).

The motivation of this paper is to present the first detailed analysis of the statistical properties of Eurofutures prices:

1. We study intraday and intraweek seasonalities in the volatility. We have found that intraday tick activity and price changes display a U-shape similar to that documented for stock indices. There is evidence of day-of-the-week effects which means that there are days which differ from the others in a statistically significant way.

2. We compare the behavior of actual trading prices and bid/ask quotes both on an intraday and intraweek basis.

3. We investigate the presence of an empirical scaling law for mean absolute price changes which shows a drift exponent significantly higher than the one produced by a pure random walk and surprisingly similar to that obtained for the foreign exchange market ((Müller et al., 1990)).

4. We show that there is a volatility seasonality depending on the time left before the contract expiry, with smaller effects related to the expiry of other Eurofutures, during the lifetime of the contract in question.

In section 2, we briefly describe the characteristics of the Eurofutures markets and the data samples under study. In section 3, we analyze data from the London International Financial Futures Exchange (LIFFE) and the Chicago Mercantile Exchange (CME) data; this includes an initial analysis of raw prices, followed by a more detailed intraday/intraweek study of tick activity and volatility for transaction prices and bid/ask quotes. In section 4, we develop our analysis per contract focusing on two important aspects: the scaling law and the deterministic volatility pattern. Section 5 focuses on the Singapore International Monetary Exchange (SIMEX) market and section 6 concludes.

2 Market and data description

Eurofutures are exchange-traded contracts; this entails several differences with respect to over-the-counter (OTC) instruments:

- They are linked to a specific exchange, except when a fungibility agreement is in effect [3].
- Trading is typically geographically localized. There is no 24-hour trading unlike for the FX market, although the trend is to effectively lengthen trading hours (e.g. with after-hours sessions).

From an operational point of view, most futures markets are characterized by two trading sessions called "in-hours" and "after-hours" trading sessions. At CME and LIFFE, in-hours sessions are represented by the traditional open outcry trading mechanism and last about 8 hours; after-hours sessions are computerized versions of open outcry and involve only trades by electronic systems. Among the two most important after-hours electronic systems are APT (Automated Pit Trading system) developed by LIFFE and GLOBEX developed by CME. For the moment,

[3] One example of a fungibility agreement is the mutual offset system between CME and SIMEX, through which contracts opened in one exchange can be liquidated on the other.

LIFFE after-hours sessions last about 90 minutes and start immediately after in-hours sessions while CME after-hours sessions change from product to product and are usually longer than LIFFE's. Compared to the open outcry trading mechanism, the electronic trading system has a different data coverage: price reporting is more accurate (there is a perfect audit trail), but liquidity is typically much lower in the after-hours sessions.

In this paper we analyze samples of tick-by-tick data collected in three different markets. In the first part we concentrate on LIFFE and CME data while in the last part we report results on SIMEX data. Given that futures contracts are exchange traded and each transaction is recorded centrally, futures markets offer a high price transparency. The historical data always includes tick-by-tick transaction prices and, depending on the data source, bid and ask quotes, bid and ask sizes and actual volumes for each transaction. This allows us to conduct the first full investigation of transaction prices compared to bid/ask quotes instead of considering only bid and ask quotes like in intraday studies of the foreign exchange market (see e.g. (Müller et al., 1990; Dacorogna et al., 1993; Guillaume et al., 1997)).

Eurofutures prices are quoted as 100 minus the corresponding forward rate. In this paper all price changes are determined by linear interpolation, unless stated otherwise [4].

Given the technical structure of futures contracts we perform two kinds of analysis: one per position and one per contract. Each futures contract has a specific expiry date and normally begins to be traded one year or more in advance. Eurofutures, referring to an underlying three month deposit, present four settlement months in a year (March, June, September and December), known as *quarterly expiries*. Sometimes there are also serial expiry contracts, (i.e., contracts expiring in months that do not correspond to the quarterly sequence), but they are not considered here, because they typically exhibit lower liquidity. We define the first position at a given time as the contract that will expire next in the quarterly sequence, the second position as the second contract to expire in the same quarterly sequence and so on. For each Eurofutures, we can build two different types of price time series:

1. A time series by position, which consists of the prices of a given position. This time series is not interrupted by contract expiries and consists of different contracts; at each quarterly expiry it switches to a new contract.
2. A time series by contract, starting on the date the contract opens for trade and stopping when the contract expires.

Our study of CME and LIFFE focuses on the following Eurofutures: Eurodollar (traded at CME), Euroswiss, Euromark, Short Sterling, Eurolira, Three Month ECU (all traded at LIFFE). We start with an analysis per position examining the last four quarters (or four positions) before expiry.

For all Eurofutures and all positions we consider a sample of more than three years: from January 1, 1994 to April 15, 1997. Then we conclude this first part with an analysis per contract performed on four Eurofutures: Eurodollar (traded at CME), Euromark, Short Sterling and Eurolira (all traded at LIFFE). For each Eurofutures we consider nine contracts associated with the following expiries: March 1995, June 1995, September 1995, December 1995, March 1996, June 1996, September 1996, December 1996 and March 1997. Considering different contracts, we have different sample lengths but, in general, we have at least one year of data for each contract.

[4] A formal definition of all the variables studied in this paper (the change of price, the volatility and the tick frequency) can be found in (Guillaume et al., 1997).

The last part of our work is devoted to an introductory study of SIMEX data. On this market, we focus our attention on Euroyen and Eurodollar futures. Both the analysis per position and per contract are performed for longer periods of time than on CME and LIFFE. For SIMEX we consider eight positions and a period longer than six years, from January 8, 1990 to May 1, 1997.

3 Analysis per position on LIFFE and CME data

We start with an analysis per position. We begin with an investigation of raw transaction prices, then we perform an hourly and a daily analysis of tick activity and transaction price changes with special attention to volatility patterns, eventually we compare hourly and weekly behavior of transaction prices and bid/ask quotes.

3.1 *Time charts*

Time charts showing the price as a function of time provide an intuitive graphical view of the main features of the time series under study. Even if there is no unique and generally accepted definition of volatility (Müller et al., 1997), time charts allow us to visualize at least two different kinds of volatility: one is the spread existing between minimum and maximum prices occurring during the considered period (long-term volatility) and the other is the daily or weekly change in prices (short-term volatility). From both points of view, all Eurofutures, except Eurolira, display an increase in volatility going from position one to position four. Position one and position four of Eurodollar futures in figure 1 represent this increase in volatility. Eurolira represents an exception for two reasons: firstly, in some intervals it displays a decreasing or invariant daily volatility when the time left before expiry increases (see again figure 1); and secondly, for all positions, it shows a wider range of price variation than for Eurofutures corresponding to all other currencies (this range tends to decrease from positions one to three and then it increases again in position four).

3.2 *Intraday and intraweek volatility seasonalities*

The intraday and intraweek statistics (IntraDW) developed at Olsen and Associates (Dacorogna et al., 1993) allows us to investigate:

1. Possible seasonalities in volatility patterns and in the level of activity.
2. Differences between the main trading session and the after-hours session.

The intraday statistics uses a uniform time grid with 24-hourly intervals while intraweek statistics uses a uniform 7-day interval grid (from Monday to Sunday) in Greenwich Mean Time (GMT). Both of them are used to measure the volatility (defined as the mean absolute value of the logarithmic price change) and the tick activity (considered as an approximation of the level of market activity) [5]. It is important to remind that this first part of our IntraDW analysis is performed entirely on transaction prices, while the following section 3.3 deals with both transaction prices and bid/ask quotes.

[5] For more details about IntraDW analysis and its variables see (Guillaume et al., 1997) and (Müller et al., 1990).

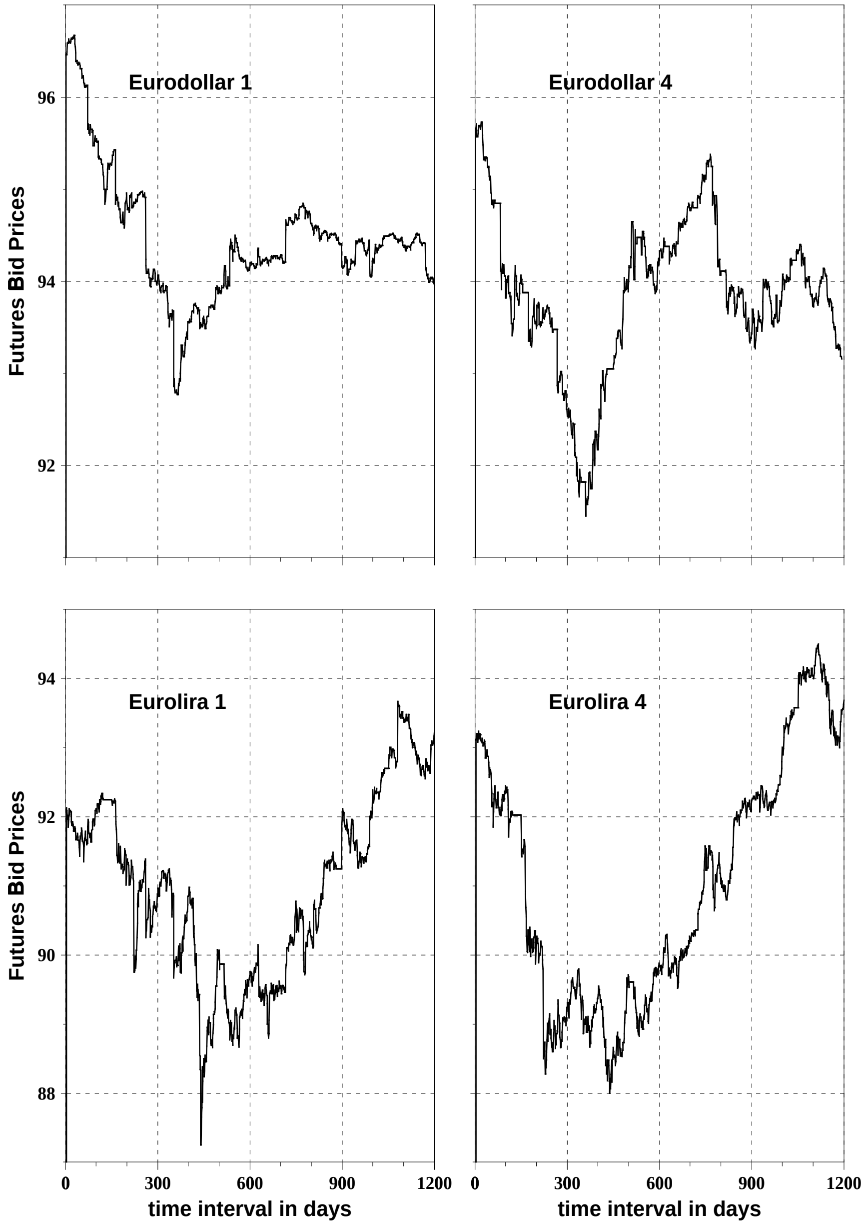

Figure 1: Time charts of Eurodollar and Eurolira prices for position one and four. The sampling period starts on January 1, 1994 and ends on April 15, 1997. The number of ticks for Eurodollar position one is 89485 and for position four is 100043. The number of ticks for Eurolira position one is 199139 and for position four is 36288. Eurodollar displays an increase in volatility from position one to four, both on short and long periods of time. It is representative of all the other Eurofutures behavior except Eurolira which displays a decreasing or invariant volatility.

To get an initial idea of the tick activity per position characterizing CME and LIFFE futures prices, we can look at Table 1 which displays the average number of ticks per business day [6]. Analyzing the results, we have to remember that during our sampling period the coverage of the electronic trading systems (APT and GLOBEX) used at LIFFE and CME had changed. This, along with the fact that these coverages are different, means that the absolute number of ticks cannot be directly compared across markets. Looking at each column of table 1, we can notice

[6] Year by year we expect an increase in the number of ticks within the considered period because of the increase in traded volumes (see the LIFFE web pages, http://www.liffe.com.).

	Eurodollar	Euromark	Sterling	Euroswiss	Eurolira	ECU
Position 1	104	161	151	84	231	41
Position 2	180	227	214	81	172	30
Position 3	151	215	158	29	74	14
Position 4	117	175	110	12	42	9

Table 1: Average number of ticks per business day, computed for each Eurofutures and each position.

two distinct behaviors:

1. Eurodollar, Euromark and Short Sterling display a high liquidity in all positions; the number of ticks reaches a maximum in the second position and then decreases. The fourth position has at most as many ticks as the first.
2. Euroswiss, Eurolira and Three Month ECU, on the contrary, display a decreasing level of liquidity from position one to position four (for Eurolira it is almost six times lower, for Euroswiss it is seven times lower); the average number of ticks for position four is really low compared to the fourth position of the previous group (for Euroswiss and Three Month ECU it is about 10 times lower). On the whole, Three Month ECU shows a limited tick activity compared to all the other Eurofutures.

Intraday tick activity and price changes show the expected intraday seasonality (see (Andersen and Bollerslev, 1997) and (Ghysels and Jasiak, 1995)). For all contracts traded on LIFFE the hourly tick activity displays a U-shape with its minimum around 11am-1pm (GMT) and a clustering of activity around the beginning and the end of the day. There are differences among Eurofutures both between the levels and widths of the peaks and the level of the decrement to the minimum. Eurodollar (traded on CME) displays a similar behavior but the activity in the first half of the working day, which takes place when the European markets are still open, is higher than during the second half of the day, when European markets have already closed, and Asian markets are not yet open.

Intraday price changes follow a pattern similar to that presented by intraday tick activity. In general, opening hours show the highest price variation (the difference with respect to the average of the other hours is around one basis point); only in some cases the largest price change occurs towards the closing time (usually in the last positions). Differences occur in some positions for Short Sterling, Eurolira and Three Month ECU which display the minimum of the U-curve one hour later than in the tick-activity case (see figure 2 which displays intraday tick activity and intraday price changes for Short Sterling in position two). The first two positions of the Euromark display less regularity in the intraday price change behavior (see figure 3 which displays intraday price changes for Euromark position one). This common behavior is confirmed also by correlation results: on the whole, the correlation between hourly tick activity and hourly price changes is above 0.96; only Euromark for the first two positions and Three Month ECU for the fourth position show a lower correlation around 0.90. In general, for Eurodollar, Euromark and Short Sterling, hourly price changes tend to increase from position one to position four; Eurodollar and

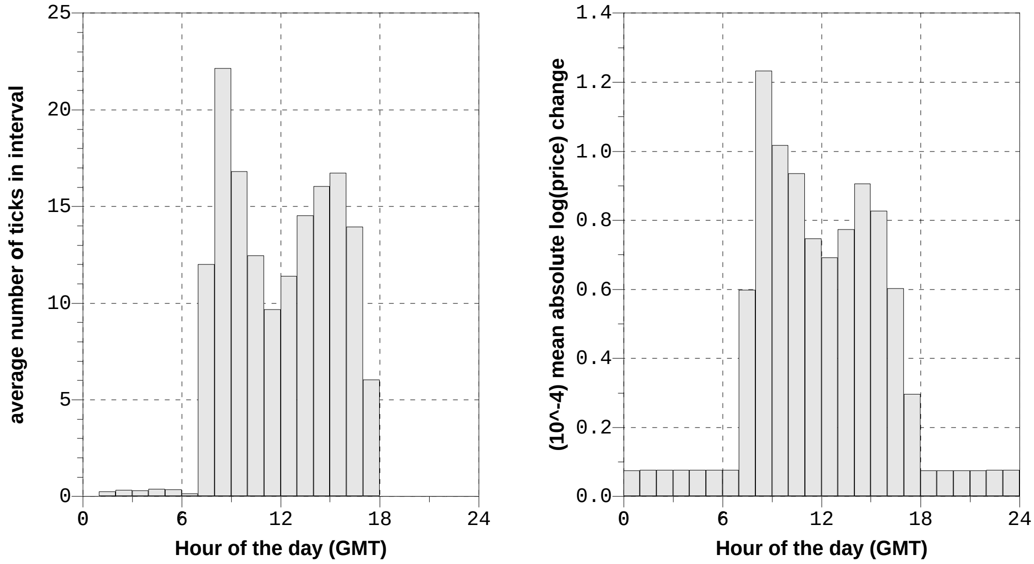

Figure 2: Intraday tick activity and intraday price changes for Short Sterling in position two. The intraday tick activity displays the average number of ticks occurring in each hour of the day while intraday price changes show the mean absolute change in the logarithm of the price. Both plots display similar U-shapes, the only difference being that the minimum appears one hour later for intraday price changes. The sampling period starts on January 1, 1994 and ends on April 15, 1997. The total number of ticks is 184360.

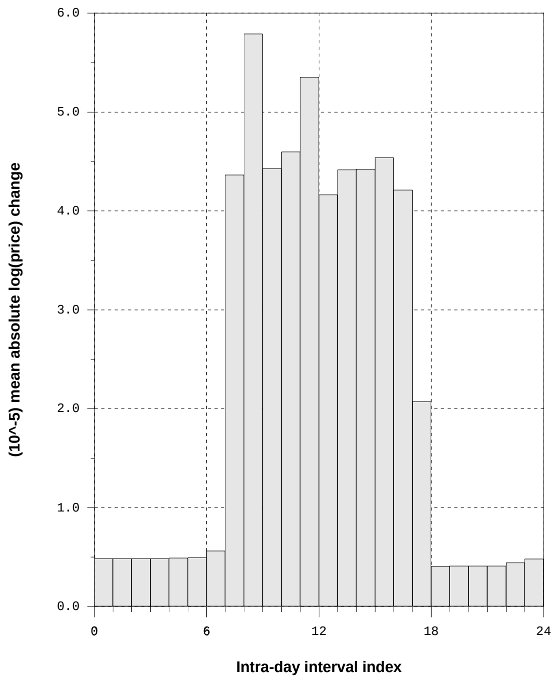

Figure 3: Intraday price changes for Euromark in position one. It displays an irregular intraday price change behavior. The sampling period starts on January 1, 1994 and ends on April 4, 1997.

Short Sterling display a decrease for some hours in position four. For Euroswiss and Three Month ECU price changes show an increment from position one to position two and then they decrease again. For Eurolira, intraday volatility has a tendency to decrease from position one to position four.

	Eurodollar		Euroswiss		Euromark		Sterling		Eurolira		ECU	
	day	week	day	week	day	week	day	week	day	week	day	week
Position 1	5.8	16.3	4.6	12.1	3.3	8.2	6.5	17.8	13.3	36.8	5.2	14.1
Position 2	6.7	18.9	5.4	13.7	4.6	11.2	7.1	19.7	11.6	32.9	5.2	13.5
Position 3	9.5	24.2	5.4	14.5	5.7	14.0	7.4	21.2	10.4	30.7	5.2	14.5
Position 4	7.1	21.0	5.2	14.9	6.1	16.2	7.6	23.0	9.8	29.9	5.4	15.8

Table 2: Daily and weekly volatility in basis points (10^{-4}) for different Eurofutures positions.

Volatility is considered as the measure of price variation but, as mentioned before, different definitions may be used. In our intraday analysis, hourly and daily volatility are defined as the mean absolute change of logarithmic prices. For completeness, we have analyzed the volatility behavior using also another definition of the volatility as the square root of the (non-annualized) variance computed with mean value. Values of daily and weekly volatility (obtained with this alternative definition), computed for each Eurofutures and each position, are reported in Table 2. We have not found significant differences between the two definitions. Results on daily volatility (see Table 2) exactly confirm the outcomes of our intraweek price change analysis: for Eurodollar, Euromark and Short Sterling the level of variation tends to increase from position one to position four (Eurodollar displays a decrease in position four); Euroswiss shows an increment from position one to position two and then starts to decrease; Three Month ECU displays a slight tendency to increase; on the contrary, for Eurolira the spread tends to decrease passing from front dates (position one) to back dates (position four). Table 2 also summarizes additional results on weekly volatility patterns which partly reflect daily outcomes. In fact, on a weekly basis, for all Eurofutures (except for Eurolira) price variations tend to increase passing from position one to position four. It is interesting to note that the ratio between the daily and weekly volatility almost always exceeds $\sqrt{7}$, the latter being the ratio predicted by the aggregation properties of the Gaussian random walk.

Looking at intraweek tick activity, there is evidence of a day-of-the-week effect. In general, the level of activity displays a minimum on Monday and a maximum on the last two working days of the week: usually on Thursday for LIFFE contracts and on Friday for CME contracts. The difference is definitely significant for the Eurodollar; in fact, for positions one and two the tick activity on Friday is almost double that on Monday and it becomes more than double for positions three and four (see for example figure 4 which shows intraweek tick activity for positions one and four). In general, there is a gradual increase from the minimum to the maximum.

The correlation between intraweek tick activity and intraweek price changes is high, but only Eurodollar intraweek price changes follow a pattern really similar to that displayed by intraweek tick activity. In fact, except for position one, the maximum price variation for Eurodollar is reached on Friday and the difference between Monday's and Friday's price changes tends to increase from positions two to four (position one shows the highest level of variation on Monday and presents the lowest value of the correlation coefficient among all the Eurofutures). In addition, the spread of price changes on Friday is on average about two basis points higher than on other days. Eurolira, for positions one and two, reaches the maximum spread on Monday and for

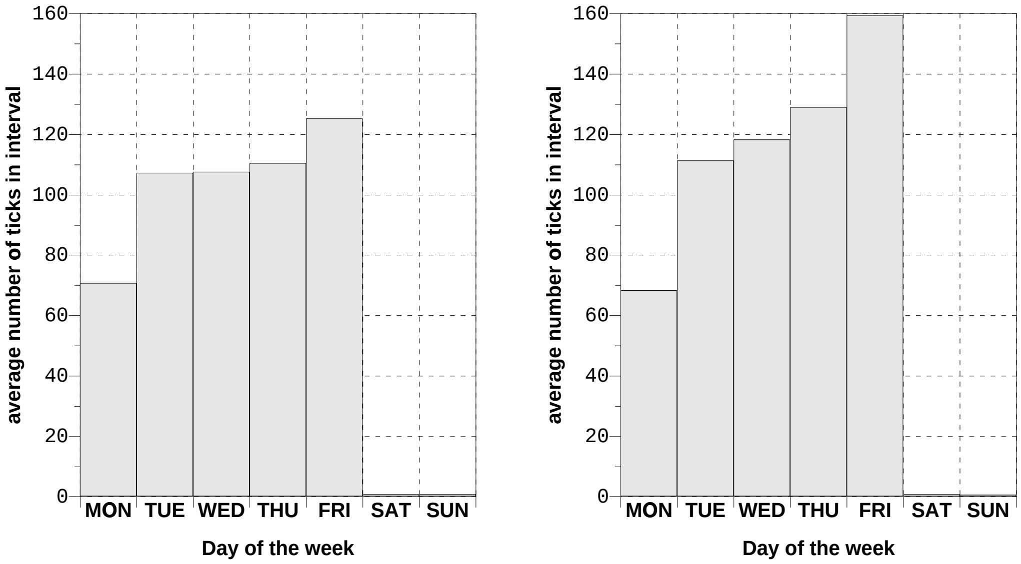

Figure 4: Intraweek tick activity for Eurodollar (CME contracts) in position one and position four. They display considerable day-of-the-week differences. The sampling period starts on January 1, 1994 and ends on April 15, 1997. The total number of ticks is 89468.

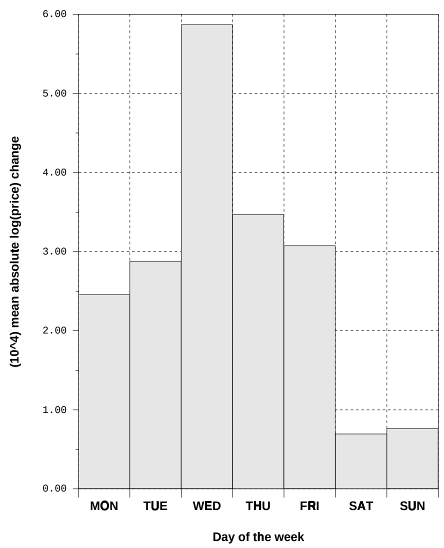

Figure 5: Intraweek price changes for Short Sterling in position one. The largest price changes occur on Wednesday. The sampling period starts on January 1, 1994 and ends on April 15, 1997.

positions three and four on the last two working days of the week. Three Month ECU, Euroswiss and Euromark reach the maximum spread on Mondays and Thursdays (usually higher on Mondays) and Short Sterling on Wednesday. In Short Sterling's case, it is worth noting that, at least for the first three positions, price changes on Wednesday are about two basis points higher than during the other days (see figure 5 in which position one results are displayed). This could be explained by the fact that the relevant news releases in the United Kingdom occur on Tuesdays and Wednesdays.

3.3 *Intraday analysis of transaction prices and bid/ask quotes*

In this section, we compare the results of our IntraDW analysis performed once on transaction prices and once on bid/ask quotes. The objective of this additional investigation is to verify if differences exist between transaction prices and bid/ask quotes and in-hours and after-hours trading sessions. This study is still performed on the same Eurofutures but on a shorter period of time, from December 2, 1996 to April 15, 1997. The comparison has highlighted the following characteristics:

- **Intraweek tick activity**: during the period under consideration, the highest level of activity in terms of bid/ask quotes is reached by Eurodollar followed by Eurolira and Short Sterling. For transaction prices, the highest level of activity is reached by Eurolira in the first two positions and by Euromark in the last two. The least-traded Eurofutures is represented by Three Month ECU from both points of view. Activity is really limited during weekends for all Eurofutures but is usually higher on Sunday and for bid/ask (with respect to transaction). During working days, two different behaviors can be identified:

  1. Eurodollar, Euroswiss and Three Month ECU display bid/ask tick activity higher than transaction activity; in particular, for Eurodollar, for all positions, tick activity for bid/ask quotes is about five times higher than tick activity for transaction prices.
  2. The contrary happens for the other Eurofutures. For Euromark activity related to transaction prices is on average double that of bid/ask quotes; the only exception is represented by Tuesday in position three (higher for bid/ask and three times higher compared to other days); for Short Sterling exceptions are represented by position four (globally, ticks associated with bid/ask quotes higher than those related to transaction prices) and by Tuesday in position two (ticks for bid/ask quotes higher than for transaction prices and three times higher than the other days as shown in figure 6); for Eurolira tick activity related to transaction prices is higher than that of bid/ask quotes only in positions one and two; in positions three and four the opposite occurs.

- **Intraweek price changes**: During business days, bid/ask price changes are usually higher than transaction ones and the difference is often less than one basis point; exceptions which display higher transaction price changes occur for all Eurofutures more often during the last two working days of the week, either on Thursdays or Fridays. Both bid/ask quotes and transaction prices confirm relevant price variations for Short Sterling (more than two basis points) on Wednesday, for Eurolira and Eurodollar on Monday for positions one and two, and almost always for Euromark on Monday. Over weekends, with very few exceptions, bid/ask price changes are higher than transaction prices, and even higher on Sunday than on Saturday. There are practically no transaction price changes on Saturday.

- **Intraday tick activity**: For all Eurofutures traded on LIFFE, intraday tick activity displays a U-shape for both transaction prices and bid/ask quotes; differences can occur because one is more asymmetric than the other or because peaks are more pronounced in one case than in the other. In general, both transaction and bid/ask tick activities are concentrated between 6-7 am and 5-6 pm (GMT time); only Euromark in position three and Short Sterling in position two display a significant number of ticks between 1 am and 6 am; this is particularly relevant for the bid/ask case. For Eurodollar hourly tick activity for bid/ask is on average four times higher than for transaction, in addition tick activity for transaction is mostly concentrated between 11 am and 9 pm (GMT) while for bid/ask there is a significant number of ticks for the entire 24 hours.

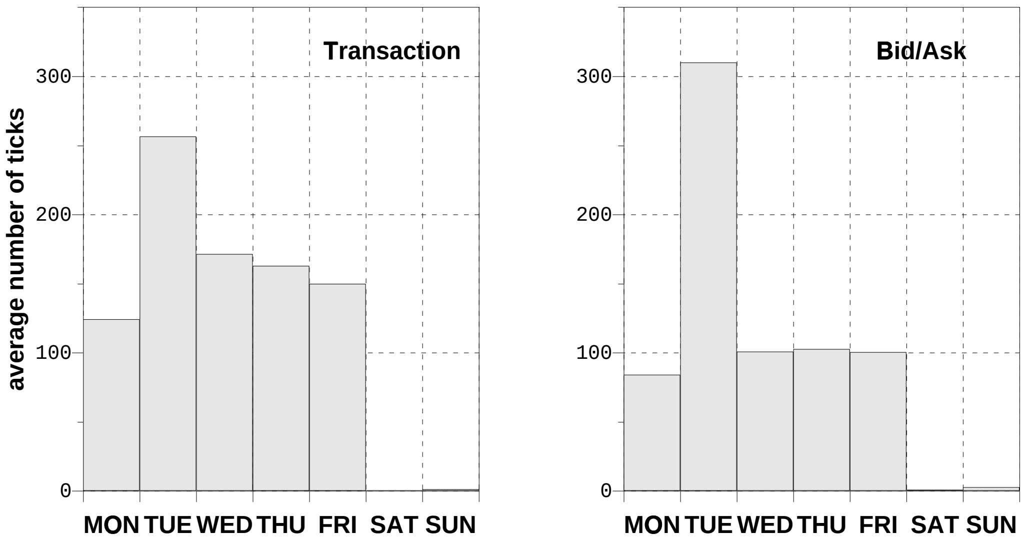

Figure 6: Intraweek tick activity for transaction prices (on the left) and intraweek tick activity for bid/ask quotes (on the right). Both are computed for Short Sterling in position two. Activity on transaction prices appears higher than activity on bid/ask quotes except on Tuesday. The sampling period starts on December 2, 1996 and ends on April 15, 1997.

- **Intraday price changes**: For both transaction prices and bid/ask quotes, all Eurofutures traded on LIFFE display significant intraday price changes between 6-7 am and 5-6 pm (GMT) (following intraday tick activity behavior). On the whole, for all positions hourly price changes are higher for bid/ask quotes than for transaction prices. For Eurodollar transaction price changes are mostly concentrated between 6 am and 1 am (GMT), while bid/ask price changes are spread over the whole day with important peaks between 12 pm and 8 pm; before 12 pm and after 8 pm price changes are always higher for bid/ask quotes than for transaction prices, instead inside that interval the contrary can happen in some hours. In general, for Eurodollar the difference between price changes established either on the basis of transaction prices or on the basis of bid/ask quotes is not so relevant between 12 pm and 8 pm (GMT); instead it tends to be bigger (from 0.2 to 0.7 basis points) outside that interval (see figure 7). That period of time corresponds to the opening hours of the market. It seems that the contrary happens for Eurofutures traded on LIFFE; before 6 am and after 6 pm there is little difference between transaction prices and bid/ask quotes while between 7 am and 6 pm the difference is more relevant but still around or less than one basis point (see figure 7).

4 Analysis per contract

The main goal of this additional investigation per contract is to deepen our knowledge about the volatility behavior of IR futures prices. As shown before, futures contracts exhibit volatility seasonalities like those reported in the literature for FX data (Müller et al., 1990) and also for equity markets ((Andersen and Bollerslev, 1997; Ghysels and Jasiak, 1995)). Those seasonalities were attributed to dealing patterns, such as market presence (Dacorogna et al., 1993). However, the characteristic feature of the futures markets when compared to FX or equity markets is that each contract has an expiry. We show that this leads to another seasonality, depending on the time left to expiry. A first impression of the relation existing between volatility and time to expiry is given by plots of raw prices. Subsequently we conduct a scaling law analysis, then a study of the deterministic volatility pattern and finally an introductory investigation of the distribution characterizing futures prices. As mentioned in section 2, this analysis of volatility structures is performed on four Eurofutures (Eurodollar, Euromark, Short Sterling, Eurolira) and for each Eurofutures on nine different contracts.

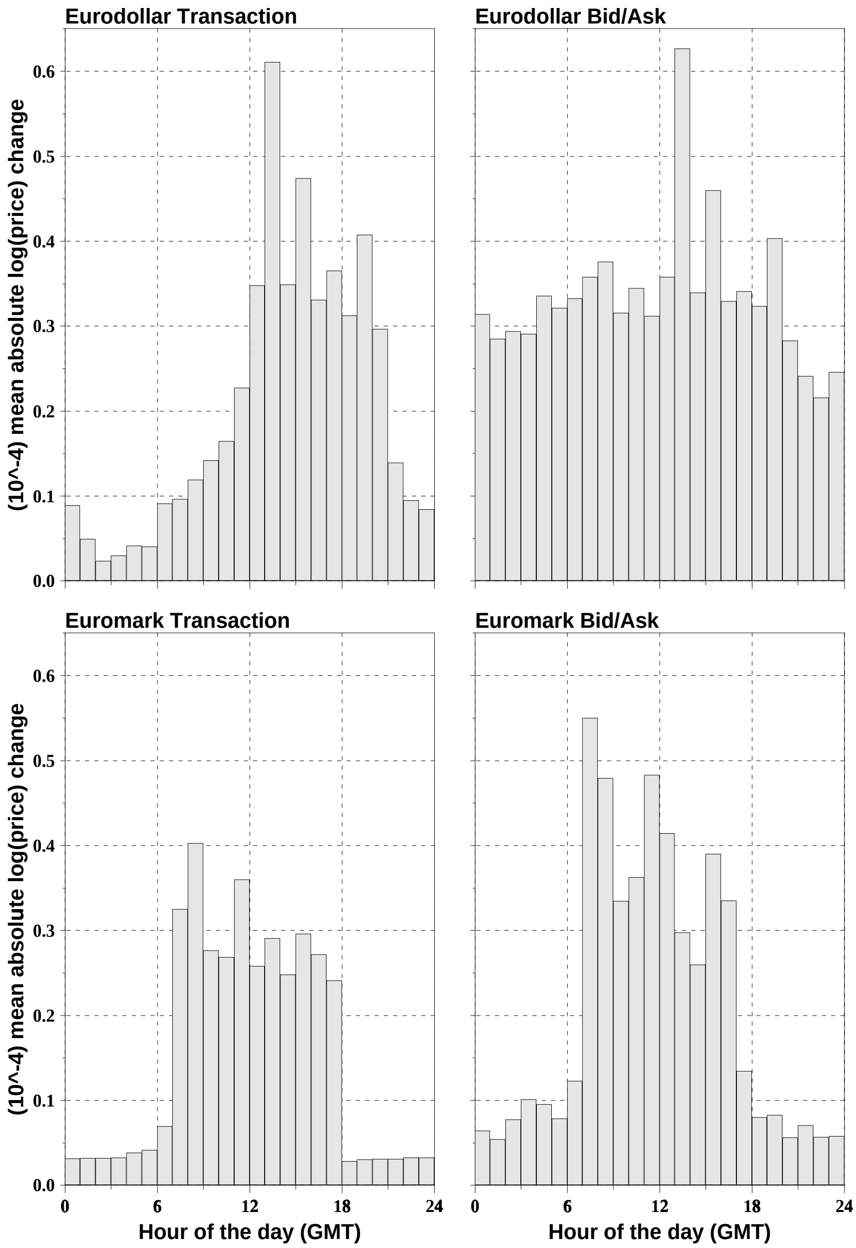

Figure 7: Hourly price changes for transaction prices (on the left) and hourly price changes for bid/ask quotes (on the right). Both are computed for Eurodollar in position one (top) and Euromark in position one (bottom). In both cases, the difference in price variation between bid/ask quotes and transaction prices is higher during open market hours than when the market is closed. The sampling period starts on December 2, 1996 and ends on April 15, 1997.

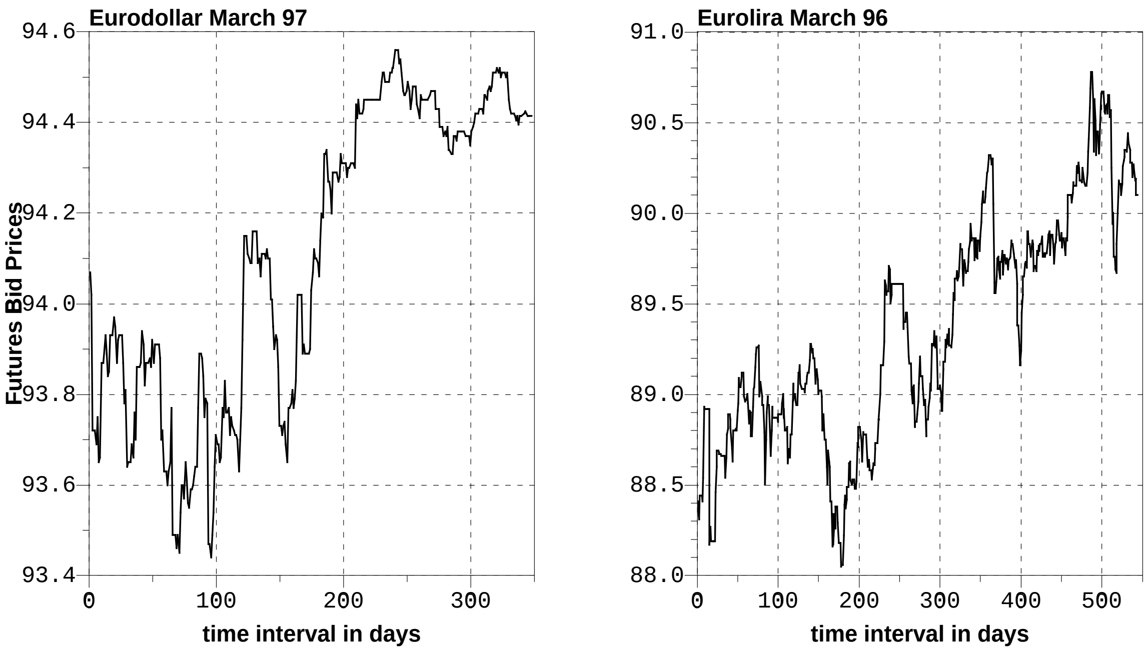

Figure 8: On the left, we plot the price of a Eurodollar contract expiring in March 1997 (sampling period: from April 3, 1994 to March 17, 1997; the number of ticks is 32591). On the right, we plot the price of a Eurolira contract expiring in March 1996 (Smpling period: from September 20, 1994, to March 3, 1996; the number of ticks is 31528).

4.1 *Time-chart description*

We have plotted futures prices against time for each Eurofutures contract. Of course such plots are not sufficient to illustrate he relation between volatility and time left before expiry. In some of them the relation is more evident (like in the case of Eurodollar: see the plot on the left side of figure 8) while in others it is more difficult to conclude (like in the case of Eurolira, shown in the plot on the right side of figure 8). On a daily or weekly basis, there are indications that Eurodollar, Euromark and Short Sterling display a decreasing volatility as we move towards expiry while Eurolira in some cases shows an increasing volatility. The different behavior seen in the Eurolira is very likely to be due to its lower liquidity, particularly for back dates.

4.2 *Scaling law of the absolute price change*

An empirical scaling law was firstly identified for foreign exchange rates (Müller et al., 1990). It provides a direct relation between intervals Δt and the average absolute price changes,

$$\overline{|\Delta x|} \quad = \quad c \; \Delta t^{\frac{1}{E}} \; , \tag{4.1}$$

where the bar over the $|\Delta x|$ indicates the average over the whole sample period. We call $1/E$ the drift exponent.

Results for drift exponents of different Eurofutures contracts, using overlapping observations according to the prescription of (Müller, 1993), are shown in table 3: the drift exponents are all significantly above the value 0.5 which is the value expected for a Gaussian random walk. The

Expiry	Eurodollar	Euromark	Sterling	Eurolira
March 1995	.60	.60	.61	.61
June 1995	.66	.65	.62	.61
September 1995	.68	.66	.62	.59
December 1995	.64	.66	.64	.54
March 1996	.57	.66	.63	.53
June 1996	.70	.62	.62	.57
September 1996	.70	.65	.62	.64
December 1996	.69	.63	.60	.65
March 1997	.66	.62	.63	.64

Table 3: Drift exponents, for all Eurofutures and all contracts. The typical error on each exponent is ≈ 0.03

time intervals we considered for the absolute returns varied from below one day to half a year.

In a second step, we have repeated the scaling law analysis on an average of contracts. We averaged the mean absolute values (associated with each time interval) on the number of contracts. When the analysis referred to single Eurofutures the average was computed on 9 contracts; when it referred to all Eurofutures and all contracts together, the average was computed on 36 contracts. Then we performed a linear regression for the logarithm of the computed averages against the corresponding logarithm of time intervals, taking the following time intervals: 1 day, 2 days, 1 week, 2 weeks, 4 weeks, 8 weeks and half a year.

The results of this additional analysis are encouraging. The scaling law seems valid for most of the considered intervals as shown in figure 9. Looking at the results (see table 4) for single Eurofutures, we note that Eurolira displays the highest level of volatility for all time intervals and the lowest value of the drift exponent. On short intervals (1 day, 2 days and 1 week), Eurolira shows almost twice the volatility of Eurodollar and Short Sterling and three times the volatility of Euromark. Short Sterling, again on short periods of time (1 and 2 days), displays a slightly higher volatility than Eurodollar, while on the remaining intervals the opposite is true; in fact, Short Sterling's drift exponent is lower than that of Eurodollar. Eurodollar and Short Sterling show higher volatility than Euromark for all intervals except the longest (corresponding to more than 26 weeks). Euromark's drift exponent is the highest among all Eurofutures. The drift exponent for all Eurofutures is remarkably similar to those obtained on FX rates (Guillaume et al., 1997) and on inter-bank money market rates (Ballocchi and Dacorogna, 1996). The fact that the scaling law for the absolute value of the returns is so consistent across different market and instruments is a remarkable empirical finding.

4.3 *Deterministic volatility*

This part of our work provides evidence that futures price volatility displays a lifetime dependence. We call deterministic volatility pattern the relation between volatility and time left to expiry. In order to probe the existence of a seasonality related to contract expiry, a sample

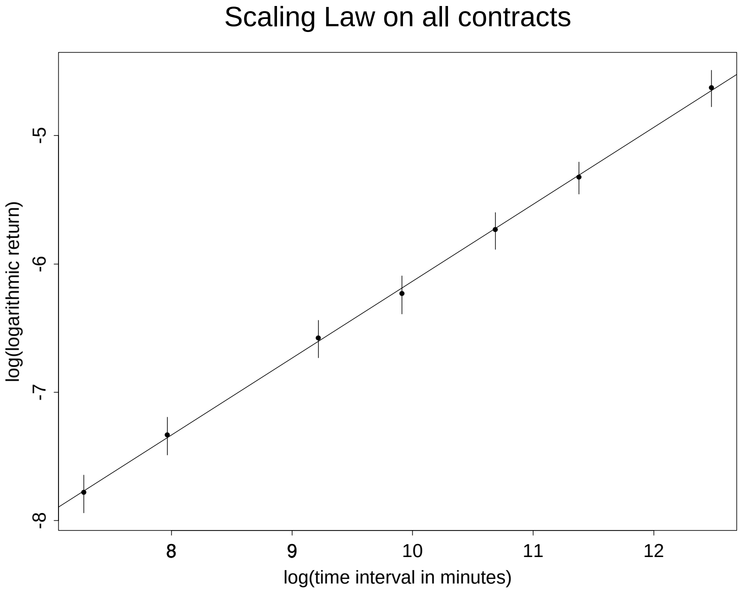

Figure 9: Scaling law over 36 short term IR futures contracts (9 for Eurodollar, 9 for Euromark, 9 for Short Sterling and 9 for Eurolira). The x-axis shows the logarithm of time intervals (from 1 day to more than 26 days, the y-axis displays the mean absolute value of the logarithmic price change computed for the average of all 36 contracts.

	All-Eurofutures	Eurodollar	Euromark	Sterling	Eurolira
drift exponent	.599	.634	.664	.599	.543
standard error	.007	.008	.023	.013	.015

Table 4: Drift exponents and standard errors for scaling law computed on Eurodollar, Euromark, Short Sterling and Eurolira separately and all together.

consisting of many futures contracts is needed.

For each Eurofutures (Eurodollar, Euromark, Short Sterling, Eurolira) and for each contract, we built a series of hourly price differences determined by linear interpolation. Then we computed daily volatilities taking the mean absolute value of hourly price differences from 00:00 to 24:00 (GMT) of each working day (weekends and holidays were excluded). Once we found daily volatilities we plotted them against time to expiry. The result is shown in figure 10. The vertical axis represents the deterministic volatility computed on all Eurofutures and all contracts together. The horizontal axis represents the time left to expiry: as we move towards the origin the number of days before expiry decreases.

Figure 10, spans a period of about 360 days because only within that period were we able to compute our deterministic volatility on at least 35 contracts. After 360 days, only a fewer number of contracts displays any real activity hence the average is affected by the volatility characterizing only few Eurofutures and it becomes less significant. The results obtained are

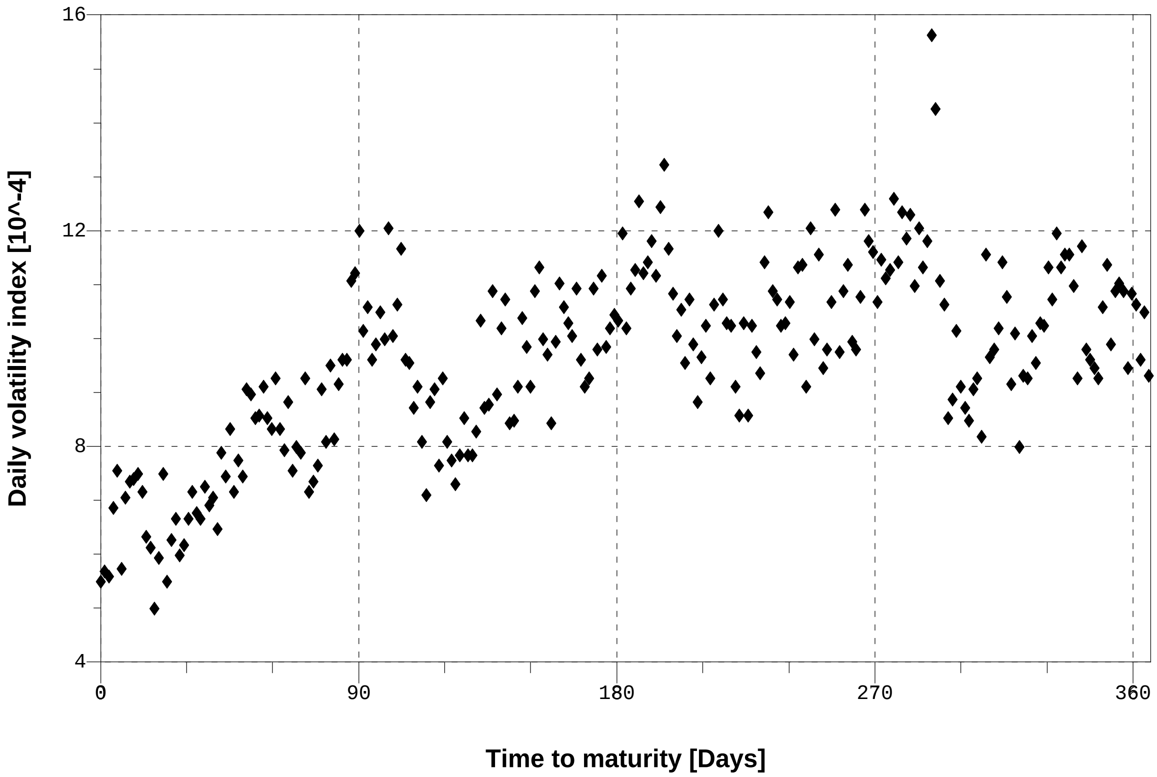

Figure 10: Volatility pattern as a function of time to expiry. The pattern is constructed as a daily average over 36 contracts (9 for Eurodollar, 9 for Euromark, 9 for Short Sterling and 9 for Eurolira). The origin is the time to expiry. The further away from the origin the further away from expiry.

quite interesting. There is a downward trend in volatility as the time left before expiry decreases: after a period of almost constant volatility between about 300 and 180 days before expiry volatility decreases faster as we move towards expiry. There is also an unexpected behavior consisting in oscillatory movements with peaks every 90 days corresponding to rollover activities near the ending of contracts. These results are confirmed also by a deterministic volatility study on each single Eurofutures. Except Eurolira, which displays an increment in volatility as we move towards expiry, Eurodollar, Euromark and Short Sterling show a decreasing volatility at least for the last 300 days before expiry. All Eurofutures (Eurolira too) display oscillatory movements with peaks around expiry dates (this appears particularly evident for Short Sterling).

4.4 *Distribution results*

We conducted an introductory study of the futures price distribution using our data per contract, with hourly price changes. We analyzed the first four moments of the distribution for each Eurofutures, each contract and for an average of contracts. The skewness of the price distribution for many contracts would seem to differ significantly from zero, using the significance level for the skewness of a Gaussian distribution, i.e. $\sqrt{15/N}$ (where N is the number of independent observations in the sample). However this is deceptive, because the price distribution in question is significantly leptokurtic. Because the sign of the skewness varies from contract to contract, with deceptively significant positive and negative values, we cannot establish conclusive evidence of non-zero skewness.

5 Analysis of SIMEX data

Most of the results found on the CME and LIFFE markets have been confirmed on SIMEX. We here present a first investigation conducted on Euromark, Eurodollar and Euroyen futures. The activity on Euromark is really limited on SIMEX when compared to Eurodollar and Euroyen. To give an example, position one for Euromark displays only 3669 ticks compared to 144861 ticks for Eurodollar and 122682 ticks for Euroyen. For Euromark, intraday tick activity and intraday price changes are mostly concentrated around settlement hours; hence activity appears to be almost entirely a settlement activity and no real liquid market exists. This shows that, apart from Eurodollar, trades on futures contracts are generally related to the geographical location of the exchange.

The following study of SIMEX Eurofutures focuses on Eurodollar and Euroyen. Plots of raw transaction prices per position reveal that for both Eurofutures:

- There is a good data coverage up to high positions.
- The total level of tick activity increases from position one to position two and then from position three onwards it starts to decrease gradually. A consistent decrease starts from position five; for instance, tick activity for position eight (about 16,000 ticks for Eurodollar and 10,000 ticks for Euroyen) is around one tenth of the activity for position one (the number of ticks of position 15 for Eurodollar is 2670).
- Volatility tends to decrease as we move closer to the expiry date.

The intraday/intraweek Euroyen seasonalities display some general characteristics:

- Intraweek tick activity for transaction prices usually reaches a maximum on Wednesday but, apart from the minimum on Monday, there is not a big difference with respect to the other days. No activity is present during weekends. Intraday tick activity is concentrated between midnight and noon (GMT), with a minimum around 3-4 am (GMT) for all positions. The tick activity exhibits an asymmetric U-shape with a maximum at open, for positions one to four. For higher positions the maximum, and in general the tick activity, shift toward the end of the trading day, indicating that most of the activity is related to the daily settlement.
- Tick activity for bid/ask quotes is generally higher than for transaction prices. For positions one to four the bid/ask tick activity [7] is twice as large as the the transaction tick activity for position one to four, and it is five times larger for positions seven and eight. The bid/ask tick activity also exhibits an asymmetric U-shape with a maximum at open, which becomes less asymmetric moving to higher futures positions, even if the shift in activity toward market close is less pronounced than for transaction activity.
- In general, intraweek price changes are higher for bid/ask quotes than for transaction prices but usually this difference is less than one basis point. The highest price variation is usually reached on Monday and the lowest on Friday; in general, the difference between minimum and maximum price changes during working days is around one basis point and is slightly higher for transaction prices. The intraday/intraweek seasonality patterns found in the tick activity are also seen in the absolute value of the hourly price changes. The difference between transaction price changes and bid/ask price changes tends to increase slightly from position one to position eight. This is due to the fact that, for transaction prices, intraday

[7] The bid/ask tick activity here is defined as the number of bid quotes plus the number of ask quotes.

price changes on average tend to decrease going to higher positions while, for bid/ask quotes, the decrease is lower.

The intraday/intraweek seasonality for the Eurodollar is characterized by a few peculiarities:

- Weekly activity always starts from a minimum on Monday and reaches a maximum on Wednesday for the first two positions, and on Friday from position three onwards (in general, there is not a big difference among the last three working days of the week). Intraday tick activity is entirely concentrated between 11 pm and 12 pm (GMT) of the next day and displays an asymmetric U-shape with minimum around 3-4 am (GMT) and maximum around 8-11 am (GMT); for all positions, most of the daily activity is concentrated after the minimum. This concentration increases position by position but, after position five, it becomes less significant because the average number of ticks per hour is less than one.
- Intraday and intraweek price changes tend to increase on average until position five and then display a slight decrease position by position. Intraday price changes display a U-shape with a minimum between 3 and 5 am (GMT). As described for LIFFE and CME, opening hours usually show the largest price change with an average of two basis points for the first positions. Hourly price changes and tick activity show a positive correlation, always above 0.8, while correlation for intraweek activity and price changes is still positive but lower, between 0.5 and 0.7. Intraweek price changes display a maximum on Monday and a minimum on Thursday or Friday; on average the difference between the maximum and the other days is greater than one basis point.

6 Conclusion

We have performed a statistical study of fundamental properties characterizing Eurofutures markets. Intra-day price changes and tick activity present a high positive correlation and display a U-shape confirming the existence of an intra-day seasonality. The activity and volatility peak at opening and closing times. There is evidence of intraweek seasonalities; in general, the level of activity displays a minimum on Monday and a maximum on the last two working days of the week (usually on Thursday for LIFFE contracts and on Friday for CME contracts). There is practically no activity during weekends.

A more detailed analysis of bid/ask quotes and transaction prices has displayed two interesting features. The first is that for only some Eurofutures (Eurodollar, Euroswiss and Three Month ECU) intraday tick activity is higher for bid/ask quotes than for transaction prices; except over weekends bid/ask tick activity prevails for all Eurofutures. The second is that the difference between hourly price changes related to transaction prices and hourly price changes related to bid/ask quotes is, in general, very small (below one basis point); for Eurodollar, the difference is almost irrelevant when the market is fully active (when the market is not active this difference increases), while for futures traded on LIFFE the difference is more important during active hours and it reaches a maximum during the first working hours of the day.

Our analysis on an average of contracts has shown the existence of a scaling law relating the absolute value of price changes to the corresponding time intervals. The scaling law displays a drift exponent significantly larger than that expected for a Gaussian random walk and very close to the values obtained for FX rates. Using again data organized per contract we have shown that price volatility displays a dependence on the time left to expiry. On average, volatility tends to decrease as we move towards expiry and, inside this main trend, it shows peaks approximately every 90 days near quarterly expiries. A first study of the futures price distribution has been conducted and has shown that price changes are distinctly leptokurtic.

A first analysis of SIMEX data shows that trade activity on Euromark futures is not really significant, confirming the expected fact that it is still rare to find a contract that trades actively in two exchanges in different continents. For Eurodollar and Euroyen, tick activity is quite high up to position four. The intraday and intraweek seasonalities at SIMEX are broadly similar to those found at LIFFE and CME: a maximum in the activity and volatility at the beginning and at the end of the day and a day-of-the-week effect (usually a minimum of activity on Monday and a maximum on Wednesday or Friday).

This study shows that the Eurofutures markets present remarkable similarities with the other markets studied so far with high frequency data (Guillaume et al., 1997; Andersen and Bollerslev, 1997; Ghysels and Jasiak, 1995), in terms of intraday/intra-week seasonalities and scaling law for absolute price changes. On the other end, we have found new properties that depend on the fact that futures contracts have predefined expiry dates. A striking example of such a property is the deterministic volatility.

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