

# An extensive set of scaling laws and the FX coastline

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## Abstract

We have discovered 17 new empirical scaling laws in foreign exchange data-series that hold for close to three orders of magnitude and across 13 currency exchange rates. Our statistical analysis crucially depends on an event-based approach that measures the relationship between different types of events. The scaling laws give an accurate estimation of the length of the price-curve coastline, which turns out to be surprisingly long. The new laws substantially extend the catalogue of stylised facts and sharply constrain the space of possible theoretical explanations of the market mechanisms.

*Keywords:* Scaling Laws; High-Frequency Finance; Foreign Exchange; Time-Series Analysis; Gaussian Random Walk Models

## 1 Introduction

The global financial system has recently been rocked by losses of 500 billion USD<sup>§</sup> and this downswing is expected to continue. The crisis is seriously undermining the functioning of the financial system, the backbone of the global economy. This suggests an acute deficiency in our understanding of how markets work. Are there “laws of nature” to be discovered in financial systems, giving us new insights? We approach this question by identifying key empirical patterns, namely scaling-law relations. We believe that these universal laws have the potential to significantly enhance our understanding of the markets.

Scaling laws establish invariance of scale and play an important role in describing complex systems (e.g. West et al. [1997], Barabási and Albert [1999], Newman [2005]). In finance, there is one scaling law that has been widely reported (Müller et al. [1990], Mantegna and Stanley [1995], Galluccio et al. [1997], Guillaume et al. [1997], Balocchi et al. [1999], Dacorogna et al. [2001], Corsi et al. [2001], Di Matteo et al. [2005]): the size of the average absolute price change (return) is scale-invariant to the time interval of its

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<sup>§</sup>E.g., [www.bloomberg.com](http://www.bloomberg.com).

occurrence. This scaling law has been applied to risk management and volatility modelling (see Ghashghaie et al. [1996], Gabaix et al. [2003], Sornette [2000], Di Matteo [2007]) even though there has been no consensus amongst researchers for why the scaling law exists (e.g., Bouchaud [2001], Barndorff-Nielsen and Prause [2001], Farmer and Lillo [2004], Lux [2006], Joulin et al. [2008]).

In the challenge of identifying new scaling laws, we analyse the price data of the foreign exchange (FX) market, a complex network made of interacting agents: corporations, institutional and retail traders, and brokers trading through market makers, who themselves form an intricate web of interdependence. With a daily turnover of more than 3 trillion USD\* and with price changes nearly every second, the FX market offers a unique opportunity to analyse the functioning of a highly liquid, over-the-counter market that is not constrained by specific exchange-based rules. In this study we consider five years of tick-by-tick data for 13 exchange rates through November 2007 (see section 3.1 for a description of the data set).

It is a common occurrence for an exchange rate to move by 10 to 20% within a year. However, since the seminal work of Mandelbrot [1963] we know about the fractal nature of price curves. The coastline at fine levels of resolution may be far longer than one might intuitively think. But how many times longer? The scaling laws described in this paper provide a surprisingly accurate estimate and highlight the importance of not only considering tail events (Sornette [2002]), but set these in perspective with the remarkably long coastline of price changes preceding them.

The remainder of the paper is organised as follows. Our main results are presented in section 2. We start by enumerating the empirical scaling laws, then cross-check our results by establishing quantitative relations amongst them and discuss the coastline. In section 3 the methods and the data are described and we conclude with some final remarks in section 4. Finally, appendix A contains tables with all the estimated scaling-law parameters.

## 2 The laws and beyond

### 2.1 The new scaling laws

Interest in scaling relations in FX data was sparked in 1990 by a seminal paper relating the mean absolute change of the logarithmic mid-prices, sampled at time intervals  $\Delta t$  over a sample of size  $n\Delta t$ , to the size of the time interval (Müller et al. [1990])

$$\langle |\Delta\chi| \rangle_p = \left( \frac{\Delta t}{C_\chi(p)} \right)^{E_\chi(p)}, \quad (0a)$$

where  $\Delta\chi_i = \chi_i - \chi_{i-1}$  and  $\chi_i = \chi(t_i) = (\ln \text{bid}_i + \ln \text{ask}_i)/2$  is the logarithmic mid-price of a currency pair at time  $t_i$ , and  $E_\chi(p)$ ,  $C_\chi(p)$  are the scaling-law parameters. The

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\*Bank for International Settlements: Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity in 2007.

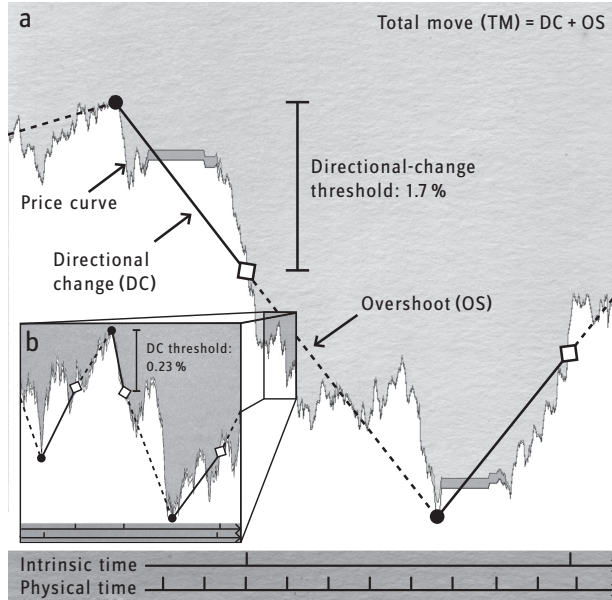


Figure 1: Projection of a (a) two-week, (b) zoomed-in 36 hour price sample onto a reduced set of so-called directional-change events defined by a threshold (a)  $\Delta x_{dc} = 1.7\%$ , (b)  $\Delta x_{dc} = 0.23\%$ . These directional-change events (diamonds) act as natural dissection points, decomposing a total-price move between two extremal price levels (bullets) into so-called directional-change (solid lines) and overshoot (dashed lines) sections. Note the increase of spread size during the two weekends with no price activity. Time scales depict physical time ticking evenly across different price-curve activity regimes, whereas *intrinsic time* triggers only at directional-change events, independent of the notion of physical time.

averaging operator is  $\langle x \rangle_p = \left( \frac{1}{n} \sum_{j=1}^n x_j^p \right)^{1/p}$ , usually with  $p \in \{1, 2\}$ , and  $p$  is omitted if equal to one. Where necessary, we employ a time interpolation scheme described in section 3.1. Throughout the paper we consider a more standard definition of the price defined as  $x_i = (\text{bid}_i + \text{ask}_i)/2$ . However, considering either  $\chi_i$  or  $x_i$  leads to very similar results even for large spread values. Later, in 1997, a second scaling law was reported by Guillaume et al. [1997], relating the number  $N(\Delta\chi_{dc})$  of so-called directional changes to the the directional-change sizes  $\Delta\chi_{dc}$

$$N(\Delta\chi_{dc}) = \left( \frac{\Delta\chi_{dc}}{C_{N,dc}} \right)^{E_{N,dc}}. \quad (0b)$$

In financial markets, the flow of time is discontinuous: over weekends trading comes to a standstill or, inversely, at news announcements there are spurts of market activity. In law (0a), the confinement of analysing returns as observed in physical time is overly restrictive. Law (0b) is a first attempt at establishing a new paradigm by looking beyond such constraints within financial data, constituting an event-driven approach, where patterns emerge for successions of events at different magnitudes. This alternative approach

## Tick-count scaling law

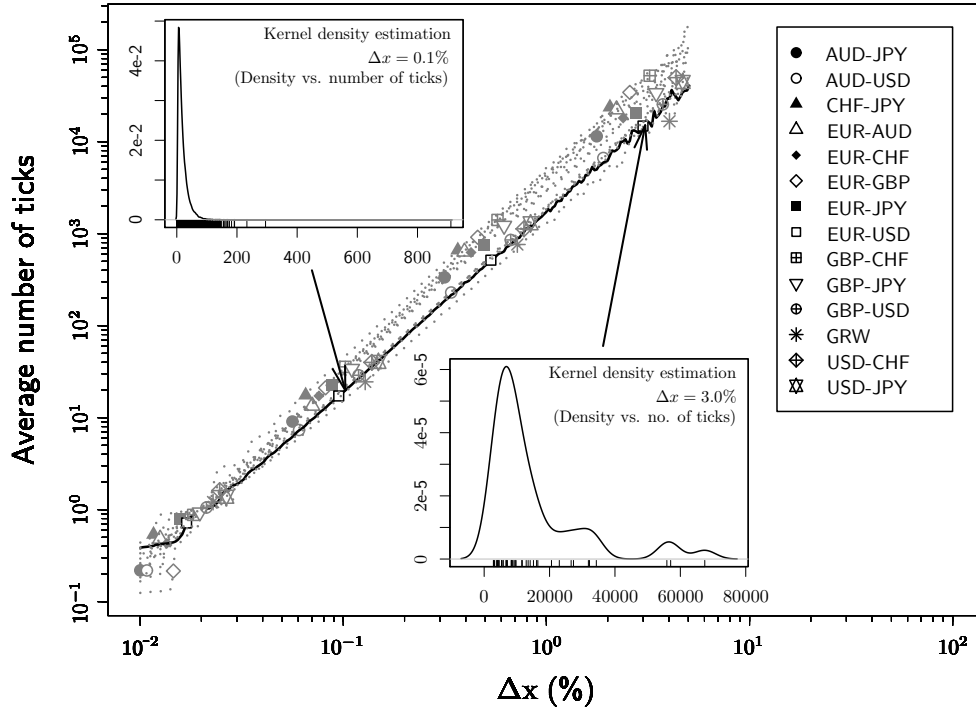


Figure 2: Scaling law (1) is plotted where the x-axis shows the price move thresholds of the observations and the y-axis the average tick numbers. A tick is defined as a price move of 0.02%. The solid line shows the raw data for EUR-USD. For the remaining 12 currency pairs and the Gaussian random walk benchmark model the raw data is displayed with dots. Insets show the distribution of the EUR-USD observations (drawn above their x-axis) for selected threshold values of 0.1% and 3.0%. See appendix A for the values of the estimated scaling-law parameters.

defines an activity-based time-scale called intrinsic time.

Extending this event-driven paradigm further enables us to observe new, stable patterns of scaling and reduces the level of complexity of real-world time series. In detail, the fixed event thresholds of different sizes define focal points, blurring out irrelevant details of the price evolution. Figure 1 depicts how the price curve is dissected into so-called directional-change and overshoot sections.

Here we confirm laws (0a) and (0b) considering  $x_i$  (see figures 3a – c), and report on 17 new scaling laws holding across 13 exchange rates and for close to three orders of magnitude. Appendix A provides tables of the estimated parameter values for all the laws and for the 13 exchange rates as well as for a Gaussian random walk (GRW) model, described in section 3.1. In addition, every table lists the average parameter values over all 13 currency pairs and their sample standard deviations. Table 1 shows the estimated scaling-law parameters for EUR-USD. We start the enumeration of the laws by a generalisation of equation (0b) that relates the average number of ticks observed during a price move of  $\Delta x$  to the size of

this threshold

$$\langle \mathbf{N}(\Delta x_{tick}) \rangle = \left( \frac{\Delta x}{C_{\mathbf{N}, tick}} \right)^{E_{\mathbf{N}, tick}}, \quad (1)$$

where a tick is defined as a price move larger than  $\Delta x_{tick} = 0.02\%$ . The definition of a tick can however be altered without destroying the scaling-law relation. Law (1) is plotted in figure 2. The second law counts the number  $\mathbf{N}(\Delta x)$  of price moves of size  $\Delta x$

$$\mathbf{N}(\Delta x) = \left( \frac{\Delta x}{C_{\mathbf{N}, x}} \right)^{E_{\mathbf{N}, x}}. \quad (2)$$

We annualise the number of observations of laws (0b) and (2) by dividing them by 5, the number of years in our data sample. Law (2) and all the following scaling laws are given in figure 3. The next scaling law relates the average maximal price move  $\Delta x_{max}$  during a time interval  $\Delta t$  to the size of that time interval

$$\langle \Delta x_{max} \rangle_p = \left( \frac{\Delta t}{C_{max}(p)} \right)^{E_{max}(p)}, \quad (3)$$

where  $\Delta x_{max} = \max\{x(\tau); \tau \in [t - \Delta t; t]\} - \min\{x(\tau); \tau \in [t - \Delta t; t]\}$  and (3) holds for  $p = 1, 2$ . We have also discovered laws relating the time during which events happen to the magnitude of these events. Law (4) relates the average time interval  $\langle \Delta t_x \rangle$  for a price change of size  $\Delta x$  to occur to the size of the threshold

$$\langle \Delta t_x \rangle = \left( \frac{\Delta x}{C_{t,x}} \right)^{E_{t,x}}, \quad (4)$$

and similarly, considering directional changes of threshold  $\Delta x_{dc}$

$$\langle \Delta t_{dc} \rangle = \left( \frac{\Delta x_{dc}}{C_{t,dc}} \right)^{E_{t,dc}}. \quad (5)$$

Next we unveil a set of scaling laws emerging from the dissection of the price curve into directional-change events (see figure 1) that make up the so-called total-move (TM) segments, which themselves decompose into directional-change (DC) and overshoot (OS) parts. The total price move, waiting time, and number of ticks can then be written as

$$\langle |\Delta x^{tm}| \rangle = \langle |\Delta x^{dc}| \rangle + \langle |\Delta x^{os}| \rangle, \quad (6)$$

$$\langle \Delta t^{tm} \rangle = \langle \Delta t^{dc} \rangle + \langle \Delta t^{os} \rangle, \quad (7)$$

$$\langle \mathbf{N}(\Delta x_{tick}^{tm}) \rangle = \langle \mathbf{N}(\Delta x_{tick}^{dc}) \rangle + \langle \mathbf{N}(\Delta x_{tick}^{os}) \rangle. \quad (8)$$

This decomposition leads to nine additional scaling laws, where the average values are

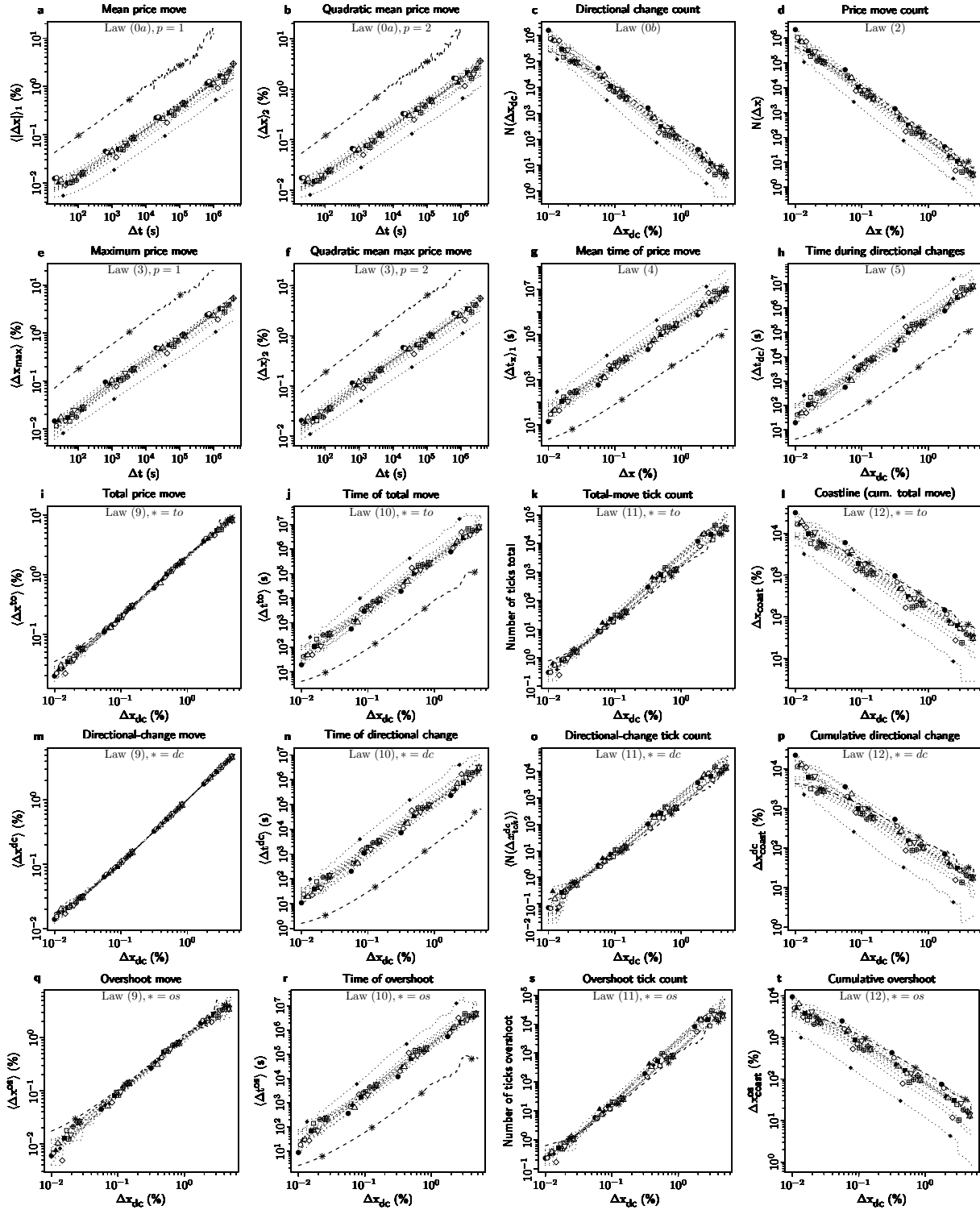


Figure 3: Plots of all scaling laws described in the text. Symbols are as in figure 2. The raw data is plotted for the 13 currency pairs with dots and for the Gaussian random walk model with dashes. See appendix A for the values of the estimated scaling-law parameters.

functions of the directional-change thresholds  $\Delta x_{dc}$

$$\langle |\Delta x^*| \rangle = \left( \frac{\Delta x_{dc}}{C_{x,*}} \right)^{E_{x,*}}, \quad (9)$$

$$\langle \Delta t^* \rangle = \left( \frac{\Delta x_{dc}}{C_{t,*}} \right)^{E_{t,*}}, \quad (10)$$

$$\langle \mathbf{N}(\Delta x_{tck}^*) \rangle = \left( \frac{\Delta x_{dc}}{C_{N,*}} \right)^{E_{N,*}}, \quad (11)$$

where  $*$  stands for  $\{tm, dc, os\}$ . Note that  $\langle |\Delta x^{dc}| \rangle = \Delta x_{dc}$  holds per definition. The average parameter values (given in appendix A) of law (9) display a peculiar feature: on average, a directional change  $\Delta x_{dc}$  is followed by an overshoot of the same magnitude ( $E_{x,to}^{av} \approx 0.99$  and  $C_{x,to}^{av} \approx 0.51$ , and  $E_{x,os}^{av} \approx 1.04$  and  $C_{x,os}^{av} \approx 1.06$ ). This result is also found by computing the probable path of the price within a binomial tree as  $0.5 \Delta x + 0.5^2 2\Delta x + 0.5^3 3\Delta x + \dots = \Delta x \sum_i^n i 0.5^i \xrightarrow{n \rightarrow \infty} 2\Delta x$ . A similar feature holds for the waiting times and number of ticks:  $\langle |\Delta t^{os}| \rangle \approx 2\langle |\Delta t^{dc}| \rangle$  and  $\langle \mathbf{N}(\Delta x_{tck}^{os}) \rangle \approx 2\langle \mathbf{N}(\Delta x_{tck}^{dc}) \rangle$ . So although in terms of size the overshoot price move is approximately as big as the direction-change threshold, it contains roughly twice as many ticks and takes twice as long to unfold.

Considering cumulative price moves instead of the averages in laws (9) leads to another triplet of laws

$$\Delta x_{cum}^* = \sum_{i=1}^n |\Delta x_i^*| = \left( \frac{\Delta x_{dc}}{C_{cum,*}} \right)^{E_{cum,*}}. \quad (12)$$

This concludes the presentation of the 17 scaling laws: we count equation (3) twice for  $p = 1, 2$ , and omit the trivial scaling law  $\langle |\Delta x^{dc}| \rangle \propto \Delta x_{dc}$ .

Our results show that most of the currency pairs exhibit similar average behaviour. This however does not appear to be true in most of the laws for EUR-CHF as seen in figure 3. Moreover, it is well-known that the statistical properties of a GRW are different from the ones observed in empirical data; Mandelbrot and Hudson [2004]. However, it is striking to observe how close this simple model can be to the average properties of the real-world data. Notable differences are seen in law (3) (see figures 3e and f) which reveals an unintuitive result: the bell-curve distribution of price moves leads to an average maximal price move that is roughly eight times larger than observed for the empirical data. In addition, although an exponent of  $1/2$  can be derived from law (0a) for  $p = 2$  considering a GRW it is incorrect to assume such an exponent for other laws. Indeed, our realisations of a GRW show exponents for all the laws but (0a) for  $p = 2$  to actually differ from  $1/2$ .

## 2.2 Exploring the space of scaling laws

The scaling laws do not represent isolated patterns but are in fact related to each other. We show that not only consistency requirements link the various laws, but that they can be combined to yield new scaling-law relations. To cross-check the laws we first compare laws

Table 1: Estimated scaling law parameter values considering EUR-USD.

| Name                          | Equation      | Table | $E$   | $C$                 |
|-------------------------------|---------------|-------|-------|---------------------|
| Tick count                    | 1             | A2    | 1.93  | $2.1 \cdot 10^{-2}$ |
| Price move count              | 2             | A3    | -1.93 | $9.5 \cdot 10^0$    |
| Maximum price move            | 3 ( $p = 1$ ) | A6    | 0.52  | $1.9 \cdot 10^5$    |
| Maximum price move            | 3 ( $p = 2$ ) | A7    | 0.49  | $1.3 \cdot 10^5$    |
| Time of price move            | 4             | A8    | 1.93  | $1.2 \cdot 10^{-3}$ |
| Time of directional change    | 5             | A9    | 1.88  | $1.1 \cdot 10^{-3}$ |
| Total price move              | 9             | A10   | 0.98  | $4.9 \cdot 10^{-1}$ |
| Overshoot move                | 9             | A12   | 1.0   | $9.9 \cdot 10^{-1}$ |
| Time of total move            | 10            | A13   | 1.89  | $1.1 \cdot 10^{-3}$ |
| Time of directional change    | 10            | A14   | 1.85  | $1.6 \cdot 10^{-3}$ |
| Time of overshoot             | 10            | A15   | 1.91  | $1.4 \cdot 10^{-3}$ |
| Total-move tick count         | 11            | A16   | 1.89  | $1.9 \cdot 10^{-2}$ |
| Directional-change tick count | 11            | A17   | 2.02  | $4.2 \cdot 10^{-2}$ |
| Overshoot tick count          | 11            | A18   | 1.87  | $2.3 \cdot 10^{-2}$ |
| Cumul. total move             | 12            | A19   | -0.94 | $2.0 \cdot 10^2$    |
| Cumul. total move w. costs    | 12            | A20   | -0.98 | $1.5 \cdot 10^2$    |
| Cumul. directional change     | 12            | A21   | -0.95 | $8.8 \cdot 10^1$    |
| Cumul. overshoot              | 12            | A22   | -0.92 | $1.1 \cdot 10^2$    |

(2) and (4), and use the fact that the average price-move time equals the sample length divided by the number of observations

$$\langle \Delta t_x \rangle = Y/N(\Delta x), \quad (13)$$

where  $Y$  is the number of seconds in a year. This implies that

$$E_{t,x} \leftrightarrow -E_{N,x} \quad \text{and} \quad C_{t,x} \leftrightarrow Y^{1/E_{N,x}} C_{N,x}. \quad (14)$$

The estimated EUR-USD parameters from table 1 allow us to verify equation (14) as  $E_{t,x} = 1.93 = -E_{N,x}$  and  $C_{t,x} = 1.23 \cdot 10^{-3} = Y^{1/E_{N,x}} C_{N,x}$ , where  $Y = 31'553'280$  seconds. Similarly, equivalent relations hold between laws (0b) and (5):  $E_{t,dc} = 1.88 \approx 1.91 = -E_{N,dc}$  and  $C_{t,dc} = 1.05 \cdot 10^{-3} \approx 1.11 \cdot 10^{-3} = Y^{1/E_{N,dc}} C_{N,dc}$ .

Furthermore, laws (0a) for  $p = 1$  and (4) are inverse relations of each other, implying

$$E_{t,x} \leftrightarrow -E_x^{-1}(1) \quad \text{and} \quad C_{t,x} \leftrightarrow C_x(1)^{-E_x(1)}. \quad (15)$$

$E_{t,x} = 1.93 \approx 2.01 = 1/E_x(1)$  and  $C_{t,x} = 1.23 \cdot 10^{-3} \approx 1.28 \cdot 10^{-3} = C_x(1)^{-E_x(1)}$ . There is a similar relationship to eq. (13) that must hold for the cumulative and dissected average moves

$$\langle |\Delta x^*| \rangle = \Delta x_{cum}^*/N(\Delta x_{dc}), \quad (16)$$

where the asterisk stands for  $\{tm, dc, os\}$ . We find for the total move with a threshold of  $\Delta x_{dc} = 0.1\%$ :  $0.2132 \approx 0.2129 = 1,244.2/5,843.4$ . Similarly, considering the directional-change and the overshoot part results in errors smaller than 0.5%. In addition,  $\Delta x_{cum}^{tm} = \Delta x_{cum}^{dc} + \Delta x_{cum}^{os}$  agrees within an error of 0.4%. The discrepancies seen in the consistency checks are in line with the fitting errors (see appendix A) and give us confidence to further proceed in exploring the space of scaling laws.

The scaling laws can be assembled to produce additional laws. As an example, laws (0a) and (1) can be used to relate the average number of ticks to a time interval  $\Delta t$

$$\langle N(\Delta x_{tck}) \rangle = \left( \frac{\Delta t}{C_x(1)C_{N,tck}^{1/E_x(1)}} \right)^{E_x(1)E_{N,tck}} = \left( \frac{\Delta t}{C_{t,tck}} \right)^{E_{t,tck}}, \quad (17)$$

where the empirical values lead to  $E_{t,tck} = 0.96$ , and  $C_{t,tck} = 279$ . This means there is a tick to be expected every 279 seconds. This expectation is compared with law (4) which indicates a move of 0.02% (i.e., a tick) every 258 seconds.

## 2.3 The coastline

We now have the necessary tools in hand to come back to the measurement of the length of the coastline. The total-move scaling law (12) allows us to estimate its size as a function of the resolution defined by the directional-change threshold. Considering thresholds of 0.01%, 0.1%, 1%, 5%, one finds the average lengths of the annualised coastline to be 22'509%, 2'046%, 186%, 34.8%, respectively. So by decreasing the threshold of resolution 500-fold, the length of the coastline decreases by a factor of 650. Similarly, looking at the GRW we find 14'361%, 1'946%, 264%, 65.2%, respectively. The 500-fold decrease in resolution entails a coastline decrease by a factor of only 220, highlighting the fact that GRW has fewer small moves and more middle-sized moves than the empirical price curves. Not surprisingly, taking transaction costs into account breaks the scaling law for small thresholds. However, it is still possible to evaluate the length of the coastline by employing the scaling relation for the interval [0.1%, 5%] and measuring it for 0.05%. Thus, for the thresholds 0.05%, 0.1%, 1%, 5% the average coastline lengths are now 1,604%, 1,463%, 161%, 34.5%. For the 0.05% threshold (which occurs on average every 15 minutes), we measure an average daily move of 6.4%. The range of these average daily coastline lengths is from 1.8% for EUR-CHF to 9.1% for AUD-JPY.

## 3 Methods and data

### 3.1 The data set

We use a tick-by-tick database composed of 13 currency pairs spanning five years, from December 1, 2002 to December 1, 2007. The following currency pairs are considered with the total number of ticks given in parenthesis: AUD-JPY (15'286'858), AUD-USD

(7'037'203), CHF-JPY (17'081'987), GBP-CHF (27'141'146), GBP-JPY (26'423'199), GBP-USD (13'918'523), EUR-AUD (19'111'129), EUR-GBP (13'847'688), EUR-CHF (9'912'921), EUR-JPY (22'594'396), EUR-USD (13'093'081), USD-CHF (13'812'055), USD-JPY (13'507'173). The difference in the number of ticks is due to varying liquidity and the fact that some exchange rates are synthetically generated from two data streams. As an example, GBP-JPY is derived from GBP-USD and USD-JPY. The data is filtered as reoccurring ticks showing the same price as the last registered tick are omitted. As the timing of price quotes does not usually coincide with the fixed sampling times implied by an interval  $\Delta t$ , for all scaling laws proportional to a power of  $\Delta t$  we use an interpolation scheme which considers the last quoted price.

In addition to the empirical data, a simple Gaussian random walk (GRW) model consisting of one million ticks is considered as a benchmark

$$\Delta x_i = x(t_i + \Delta t) - x(t_i) \sim \mathcal{N}(0, \sigma^2), \quad (18)$$

where  $x_0 = 1.336723$ ,  $\Delta t = 1$  second, and  $\sigma = 1/6769.6$ . This setting is arbitrarily chosen so as to mimic realistic FX behaviour.

## 3.2 Data fitting

It is worth noting that we do not attempt to fit power-law distributions to empirical data (see Clauset et al. [2007]). We actually make no claim on how the data is distributed for each predefined point of observation. As seen in the insets of figure 2 it is unclear to what family of distributions they belong. Rather, we detect scaling-law relations for the average and cumulative values of various quantities uncovered in the empirical data.

There are 250 data points for laws proportional to price thresholds, and the range is from 0.01% to 5.05% in logarithmic steps: in log-space the difference of the threshold values is always 0.025. For laws depending on time intervals, there are 245 observations, and the range is from 20 to 3'975'783 seconds (which is 46 days, 23 minutes and 3 seconds). The logarithmic steps are always 0.05.

We assume a linear relationship between the response variable  $Y$  and the random variables  $X$ , or  $Y = A + BX$ , where  $A$  and  $B$  are the unknown parameters to be estimated. The actual fitting is done using the  $R$  programming language's<sup>†</sup> linear model function. In addition to the linear model, a quadratic model is tested as an alternative hypothesis, i.e.,  $Y = A + BX + CX^2$ , to detect systematic curvature in the fitted data.

The tables provided in appendix A give the estimated scaling-law parameters for all 13 currency pairs and a GRW model, plus their errors. In addition, we report the adjusted  $R^2$  values of the fits. In the last column we check for any curvature by comparing the quadratic model's  $R^2$  value to the previously reported linear value, i.e.,  $R_{quad}^2 - R_{lin}^2$ . The quadratic model yields mostly an improved fit for the GRW data. The last row shows the average parameter values for the currency pairs and in parentheses the sample standard deviations.

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<sup>†</sup>[www.r-project.org](http://www.r-project.org).

From the linear model, it is straightforward to retrieve a scaling law relation:

$$y = \left(\frac{x}{C}\right)^E, \quad (19)$$

where  $y = \exp Y$ ,  $x = \exp X$ ,  $E = B$ , and  $C = \exp(-A/B)$ . To see how the error of  $C$  propagates, we assume  $A, B \sim \mathcal{N}(\mu_{A,B}, \sigma_{A,B}^2)$  and use the approximation

$$\text{Var}[C] \approx \left(\frac{\partial f}{\partial A}\bigg|_{\mu_A, \mu_B} \sigma_A\right)^2 + \left(\frac{\partial f}{\partial B}\bigg|_{\mu_A, \mu_B} \sigma_B\right)^2, \quad (20)$$

where  $f(A, B) = \exp(-A/B)$ .

## 4 Conclusions

We have enlarged the catalogue of FX stylised facts by observing 17 new scaling laws holding for close to three orders of magnitude and across 13 currency pairs. Our analysis relied heavily on understanding the empirical time series as an event-based process, instead of focusing on their stochastic nature. Relationships amongst the scaling laws can be derived and combinations of them yield new laws. Considering an 0.05% threshold, and taking costs into account, the coastline measures on average 6.4% *per day*. This is astonishingly long, and, to our knowledge, has not been mentioned in the literature. In contrast, on average across all currency pairs there is a mean maximal move of 0.60% to be observed within 24 hours (law (3),  $E_{max}^{av}(1)$ ,  $C_{max}^{av}(1)$ ), and on average it takes 220 days for a move of 6.4% to be measured (law (4)). This indicates the importance of considering not only the tail events associated with crisis, but also accounting for the numerous smaller events that precede it.

In finance, where frames of reference and fixed points are hard to come by and often illusory, the new scaling laws provide a reliable framework. We believe they can enhance our study of the dynamic behaviour of markets and improve the quality of the inferences and predictions we make about the behaviour of prices. The new laws represent the foundation of a completely new generation of tools for studying volatility, measuring risk, and creating better forecasting and trading models.

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# A Tables

Table A1: Directional change count, law (0b)

| Currency         | $E_{N,dc}$ | $\Delta E_{N,dc}$ | $C_{N,dc}$ | $\Delta C_{N,dc}$ | Adj. $R^2$ | $R^2_{quad} - R^2_{lin}$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | -2.046     | $\pm 4.6e-03$     | 1.116e+01  | $\pm 8.3e-02$     | 0.99877    | 2.775e-04                |
| AUD-USD          | -1.949     | $\pm 4.4e-03$     | 1.136e+01  | $\pm 8.7e-02$     | 0.99873    | 4.807e-04                |
| CHF-JPY          | -2.067     | $\pm 4.7e-03$     | 8.699e+00  | $\pm 6.3e-02$     | 0.99871    | 1.068e-04                |
| EUR-AUD          | -2.133     | $\pm 5.8e-03$     | 8.233e+00  | $\pm 7.0e-02$     | 0.99818    | 6.540e-05                |
| EUR-CHF          | -2.158     | $\pm 5.4e-03$     | 3.218e+00  | $\pm 2.1e-02$     | 0.99844    | 2.642e-04                |
| EUR-GBP          | -2.178     | $\pm 7.6e-03$     | 5.430e+00  | $\pm 5.5e-02$     | 0.99696    | 1.004e-03                |
| EUR-JPY          | -2.002     | $\pm 2.9e-03$     | 9.083e+00  | $\pm 4.2e-02$     | 0.99949    | 1.090e-04                |
| EUR-USD          | -1.908     | $\pm 5.0e-03$     | 9.422e+00  | $\pm 8.1e-02$     | 0.99827    | 1.031e-03                |
| GBP-CHF          | -2.131     | $\pm 3.1e-03$     | 6.406e+00  | $\pm 2.7e-02$     | 0.99949    | 4.152e-05                |
| GBP-JPY          | -2.017     | $\pm 3.4e-03$     | 9.440e+00  | $\pm 5.1e-02$     | 0.99931    | -2.733e-06               |
| GBP-USD          | -1.904     | $\pm 3.2e-03$     | 8.947e+00  | $\pm 4.8e-02$     | 0.99931    | 2.562e-04                |
| GRW              | -1.797     | $\pm 9.3e-03$     | 1.528e+01  | $\pm 2.8e-01$     | 0.99337    | 5.137e-03                |
| USD-CHF          | -1.908     | $\pm 3.3e-03$     | 1.070e+01  | $\pm 6.1e-02$     | 0.99928    | 3.056e-04                |
| USD-JPY          | -1.928     | $\pm 4.6e-03$     | 9.841e+00  | $\pm 7.7e-02$     | 0.99857    | 7.382e-04                |
| Currency average | -2.03      | (1.0e-01)         | 8.61e+00   | (2.3e+00)         |            |                          |

Table A2: Tick count, law (1),  $\Delta x_{tck} = 0.02\%$

| Currency         | $E_{N,tck}$ | $\Delta E_{N,tck}$ | $C_{N,tck}$ | $\Delta C_{N,tck}$ | Adj. $R^2$ | $R^2_{quad} - R^2_{lin}$ |
|------------------|-------------|--------------------|-------------|--------------------|------------|--------------------------|
| AUD-JPY          | 2.051       | $\pm 4.0e-03$      | 1.879e-02   | $\pm 1.7e-04$      | 0.99906    | 4.957e-05                |
| AUD-USD          | 1.970       | $\pm 3.8e-03$      | 2.146e-02   | $\pm 1.9e-04$      | 0.99906    | -3.722e-06               |
| CHF-JPY          | 2.085       | $\pm 5.4e-03$      | 1.663e-02   | $\pm 2.0e-04$      | 0.99833    | 1.392e-04                |
| EUR-AUD          | 2.134       | $\pm 4.2e-03$      | 2.028e-02   | $\pm 1.8e-04$      | 0.99904    | 1.879e-04                |
| EUR-CHF          | 2.120       | $\pm 5.2e-03$      | 2.053e-02   | $\pm 2.3e-04$      | 0.99848    | 7.283e-04                |
| EUR-GBP          | 2.185       | $\pm 9.4e-03$      | 2.150e-02   | $\pm 4.2e-04$      | 0.99541    | 1.699e-03                |
| EUR-JPY          | 1.997       | $\pm 3.0e-03$      | 1.864e-02   | $\pm 1.3e-04$      | 0.99946    | 1.960e-04                |
| EUR-USD          | 1.928       | $\pm 3.2e-03$      | 2.099e-02   | $\pm 1.6e-04$      | 0.99933    | 1.827e-04                |
| GBP-CHF          | 2.122       | $\pm 3.0e-03$      | 1.931e-02   | $\pm 1.3e-04$      | 0.99950    | 1.265e-04                |
| GBP-JPY          | 2.027       | $\pm 2.7e-03$      | 1.920e-02   | $\pm 1.2e-04$      | 0.99955    | -1.793e-06               |
| GBP-USD          | 1.932       | $\pm 2.2e-03$      | 2.035e-02   | $\pm 1.1e-04$      | 0.99967    | 6.484e-05                |
| GRW              | 1.864       | $\pm 6.0e-03$      | 2.112e-02   | $\pm 3.1e-04$      | 0.99740    | 1.838e-03                |
| USD-CHF          | 1.945       | $\pm 3.2e-03$      | 2.027e-02   | $\pm 1.5e-04$      | 0.99932    | 2.199e-04                |
| USD-JPY          | 1.975       | $\pm 3.9e-03$      | 2.206e-02   | $\pm 2.0e-04$      | 0.99901    | 4.497e-04                |
| Currency average | -2.04       | (8.6e-02)          | 2.0e-02     | (1.5e-03)          |            |                          |

Table A3: Price move count, law (2)

| Currency         | $E_{N,x}$ | $\Delta E_{N,x}$     | $C_{N,x}$ | $\Delta C_{N,x}$     | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|-----------|----------------------|-----------|----------------------|------------|--------------------------|
| AUD-JPY          | -2.051    | $\pm 4.0\text{e-}03$ | 1.111e+01 | $\pm 7.2\text{e-}02$ | 0.99907    | 4.533e-05                |
| AUD-USD          | -1.973    | $\pm 3.8\text{e-}03$ | 1.125e+01 | $\pm 7.3\text{e-}02$ | 0.99907    | -2.901e-06               |
| CHF-JPY          | -2.092    | $\pm 5.4\text{e-}03$ | 8.095e+00 | $\pm 6.5\text{e-}02$ | 0.99837    | 8.115e-05                |
| EUR-AUD          | -2.135    | $\pm 4.1\text{e-}03$ | 7.912e+00 | $\pm 4.8\text{e-}02$ | 0.99907    | 1.727e-04                |
| EUR-CHF          | -2.138    | $\pm 4.8\text{e-}03$ | 3.171e+00 | $\pm 1.9\text{e-}02$ | 0.99875    | 3.725e-04                |
| EUR-GBP          | -2.187    | $\pm 9.3\text{e-}03$ | 5.141e+00 | $\pm 6.3\text{e-}02$ | 0.99547    | 1.638e-03                |
| EUR-JPY          | -2.001    | $\pm 2.8\text{e-}03$ | 9.137e+00 | $\pm 4.1\text{e-}02$ | 0.99951    | 1.469e-04                |
| EUR-USD          | -1.930    | $\pm 3.2\text{e-}03$ | 9.469e+00 | $\pm 5.1\text{e-}02$ | 0.99932    | 2.033e-04                |
| GBP-CHF          | -2.124    | $\pm 3.0\text{e-}03$ | 6.210e+00 | $\pm 2.6\text{e-}02$ | 0.99951    | 1.162e-04                |
| GBP-JPY          | -2.029    | $\pm 2.8\text{e-}03$ | 9.142e+00 | $\pm 4.1\text{e-}02$ | 0.99953    | -3.668e-07               |
| GBP-USD          | -1.936    | $\pm 2.3\text{e-}03$ | 8.758e+00 | $\pm 3.3\text{e-}02$ | 0.99965    | 9.540e-05                |
| GRW              | -1.866    | $\pm 6.1\text{e-}03$ | 1.419e+01 | $\pm 1.6\text{e-}01$ | 0.99737    | 1.907e-03                |
| USD-CHF          | -1.946    | $\pm 3.2\text{e-}03$ | 1.022e+01 | $\pm 5.6\text{e-}02$ | 0.99931    | 2.329e-04                |
| USD-JPY          | -1.978    | $\pm 4.0\text{e-}03$ | 9.048e+00 | $\pm 5.9\text{e-}02$ | 0.99897    | 4.988e-04                |
| Currency average | -2.04     | (8.8e-02)            | 8.36e+00  | (2.3e+00)            |            |                          |

Table A4: Mean price move during  $\Delta t$ , law (0a),  $p = 1$ 

| Currency         | $E_x(1)$ | $\Delta E_x(1)$      | $C_x(1)$  | $\Delta C_x(1)$      | Adj. $R_{lin}^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|----------|----------------------|-----------|----------------------|------------------|--------------------------|
| AUD-JPY          | 0.462    | $\pm 1.3\text{e-}03$ | 4.783e+05 | $\pm 2.2\text{e+}04$ | 0.99809          | 1.261e-03                |
| AUD-USD          | 0.473    | $\pm 1.8\text{e-}03$ | 4.779e+05 | $\pm 3.0\text{e+}04$ | 0.99646          | 2.108e-03                |
| CHF-JPY          | 0.461    | $\pm 1.1\text{e-}03$ | 7.962e+05 | $\pm 3.1\text{e+}04$ | 0.99872          | 6.185e-05                |
| EUR-AUD          | 0.451    | $\pm 1.3\text{e-}03$ | 8.584e+05 | $\pm 4.1\text{e+}04$ | 0.99802          | 1.233e-03                |
| EUR-CHF          | 0.450    | $\pm 2.0\text{e-}03$ | 6.362e+06 | $\pm 5.1\text{e+}05$ | 0.99538          | 3.115e-03                |
| EUR-GBP          | 0.456    | $\pm 1.8\text{e-}03$ | 1.825e+06 | $\pm 1.2\text{e+}05$ | 0.99630          | 2.198e-03                |
| EUR-JPY          | 0.483    | $\pm 7.6\text{e-}04$ | 6.899e+05 | $\pm 1.8\text{e+}04$ | 0.99940          | 1.606e-04                |
| EUR-USD          | 0.497    | $\pm 1.1\text{e-}03$ | 6.632e+05 | $\pm 2.5\text{e+}04$ | 0.99875          | 7.131e-04                |
| GBP-CHF          | 0.462    | $\pm 8.8\text{e-}04$ | 1.317e+06 | $\pm 4.3\text{e+}04$ | 0.99913          | 3.825e-04                |
| GBP-JPY          | 0.476    | $\pm 7.7\text{e-}04$ | 6.637e+05 | $\pm 1.8\text{e+}04$ | 0.99936          | 2.521e-04                |
| GBP-USD          | 0.496    | $\pm 9.9\text{e-}04$ | 7.517e+05 | $\pm 2.5\text{e+}04$ | 0.99903          | 5.506e-04                |
| GRW              | 0.510    | $\pm 3.1\text{e-}03$ | 1.070e+04 | $\pm 8.4\text{e+}02$ | 0.99200          | 8.416e-04                |
| USD-CHF          | 0.493    | $\pm 9.6\text{e-}04$ | 5.516e+05 | $\pm 1.8\text{e+}04$ | 0.99907          | 2.802e-04                |
| USD-JPY          | 0.482    | $\pm 9.3\text{e-}04$ | 6.949e+05 | $\pm 2.2\text{e+}04$ | 0.99909          | 3.420e-04                |
| Currency average | 0.47     | (1.7e-02)            | 1.24e+06  | (1.6e+06)            |                  |                          |

Table A5: Quadratic mean price move (historical volatility) during  $\Delta t$ , law (0a),  $p = 2$ 

| Currency         | $E_x(2)$ | $\Delta E_x(2)$      | $C_x(2)$  | $\Delta C_x(2)$      | Adj. $R_{lin}^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|----------|----------------------|-----------|----------------------|------------------|--------------------------|
| AUD-JPY          | 0.454    | $\pm 9.5\text{e-}04$ | 2.274e+05 | $\pm 7.5\text{e+}03$ | 0.99892          | 3.195e-04                |
| AUD-USD          | 0.458    | $\pm 1.2\text{e-}03$ | 2.616e+05 | $\pm 1.1\text{e+}04$ | 0.99822          | 8.464e-04                |
| CHF-JPY          | 0.449    | $\pm 8.6\text{e-}04$ | 4.225e+05 | $\pm 1.3\text{e+}04$ | 0.99910          | -2.042e-06               |
| EUR-AUD          | 0.441    | $\pm 1.1\text{e-}03$ | 4.467e+05 | $\pm 1.8\text{e+}04$ | 0.99852          | 8.995e-04                |
| EUR-CHF          | 0.430    | $\pm 1.4\text{e-}03$ | 3.888e+06 | $\pm 2.3\text{e+}05$ | 0.99727          | 1.685e-03                |
| EUR-GBP          | 0.440    | $\pm 1.5\text{e-}03$ | 1.018e+06 | $\pm 5.7\text{e+}04$ | 0.99730          | 1.473e-03                |
| EUR-JPY          | 0.467    | $\pm 6.3\text{e-}04$ | 3.852e+05 | $\pm 8.4\text{e+}03$ | 0.99955          | -1.412e-06               |
| EUR-USD          | 0.473    | $\pm 7.6\text{e-}04$ | 3.843e+05 | $\pm 1.0\text{e+}04$ | 0.99936          | 6.888e-05                |
| GBP-CHF          | 0.450    | $\pm 6.3\text{e-}04$ | 7.212e+05 | $\pm 1.7\text{e+}04$ | 0.99951          | 7.869e-05                |
| GBP-JPY          | 0.463    | $\pm 6.2\text{e-}04$ | 3.588e+05 | $\pm 7.8\text{e+}03$ | 0.99956          | 5.572e-06                |
| GBP-USD          | 0.475    | $\pm 6.8\text{e-}04$ | 4.399e+05 | $\pm 1.0\text{e+}04$ | 0.99950          | 9.130e-05                |
| GRW              | 0.500    | $\pm 2.3\text{e-}03$ | 7.133e+03 | $\pm 4.1\text{e+}02$ | 0.99548          | 1.762e-05                |
| USD-CHF          | 0.472    | $\pm 7.6\text{e-}04$ | 3.131e+05 | $\pm 8.1\text{e+}03$ | 0.99937          | -1.410e-06               |
| USD-JPY          | 0.463    | $\pm 6.5\text{e-}04$ | 3.912e+05 | $\pm 8.9\text{e+}03$ | 0.99952          | 3.957e-05                |
| Currency average | 0.46     | (1.4e-02)            | 7.12e+05  | (9.8e+05)            |                  |                          |

Table A6: Maximal price move during  $\Delta t$ , law (3),  $p = 1$ 

| Currency         | $E_{max}(1)$ | $\Delta E_{max}(1)$  | $C_{max}(1)$ | $\Delta C_{max}(1)$  | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|--------------|----------------------|--------------|----------------------|------------|--------------------------|
| AUD-JPY          | 0.479        | $\pm 1.1\text{e-}03$ | 9.743e+04    | $\pm 3.4\text{e+}03$ | 0.99867    | 5.190e-04                |
| AUD-USD          | 0.511        | $\pm 1.2\text{e-}03$ | 1.106e+05    | $\pm 3.9\text{e+}03$ | 0.99867    | 8.429e-04                |
| CHF-JPY          | 0.478        | $\pm 7.1\text{e-}04$ | 1.515e+05    | $\pm 3.4\text{e+}03$ | 0.99947    | 3.451e-04                |
| EUR-AUD          | 0.464        | $\pm 1.1\text{e-}03$ | 1.523e+05    | $\pm 5.4\text{e+}03$ | 0.99872    | 3.016e-04                |
| EUR-CHF          | 0.466        | $\pm 4.2\text{e-}04$ | 1.084e+06    | $\pm 1.7\text{e+}04$ | 0.99980    | 7.757e-06                |
| EUR-GBP          | 0.467        | $\pm 4.6\text{e-}04$ | 3.410e+05    | $\pm 5.4\text{e+}03$ | 0.99977    | 4.654e-06                |
| EUR-JPY          | 0.495        | $\pm 7.8\text{e-}04$ | 1.572e+05    | $\pm 3.8\text{e+}03$ | 0.99939    | 2.617e-04                |
| EUR-USD          | 0.521        | $\pm 1.1\text{e-}03$ | 1.676e+05    | $\pm 5.7\text{e+}03$ | 0.99884    | 8.795e-04                |
| GBP-CHF          | 0.469        | $\pm 7.2\text{e-}04$ | 2.528e+05    | $\pm 6.1\text{e+}03$ | 0.99942    | 1.830e-04                |
| GBP-JPY          | 0.487        | $\pm 9.5\text{e-}04$ | 1.426e+05    | $\pm 4.3\text{e+}03$ | 0.99908    | 3.303e-04                |
| GBP-USD          | 0.522        | $\pm 1.2\text{e-}03$ | 1.873e+05    | $\pm 6.5\text{e+}03$ | 0.99880    | 8.679e-04                |
| GRW              | 0.513        | $\pm 1.0\text{e-}03$ | 2.996e+03    | $\pm 7.2\text{e+}01$ | 0.99914    | 6.057e-05                |
| USD-CHF          | 0.516        | $\pm 1.1\text{e-}03$ | 1.355e+05    | $\pm 4.4\text{e+}03$ | 0.99893    | 8.976e-04                |
| USD-JPY          | 0.508        | $\pm 1.2\text{e-}03$ | 1.579e+05    | $\pm 5.7\text{e+}03$ | 0.99865    | 1.020e-03                |
| Currency average | 0.49         | (2.2e-02)            | 2.41e+05     | (2.6e+05)            |            |                          |

Table A7: Quadratic maximal price move during  $\Delta t$ , law (3),  $p = 2$ 

| Currency         | $E_{max}(2)$ | $\Delta E_{max}(2)$ | $C_{max}(2)$ | $\Delta C_{max}(2)$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|--------------|---------------------|--------------|---------------------|------------|--------------------------|
| AUD-JPY          | 0.470        | $\pm 7.5e-04$       | 6.850e+04    | $\pm 1.6e+03$       | 0.99939    | 2.267e-04                |
| AUD-USD          | 0.486        | $\pm 7.5e-04$       | 8.513e+04    | $\pm 2.0e+03$       | 0.99942    | 2.839e-04                |
| CHF-JPY          | 0.466        | $\pm 6.0e-04$       | 1.198e+05    | $\pm 2.4e+03$       | 0.99959    | 2.618e-04                |
| EUR-AUD          | 0.450        | $\pm 6.7e-04$       | 1.191e+05    | $\pm 2.7e+03$       | 0.99947    | 2.459e-05                |
| EUR-CHF          | 0.446        | $\pm 3.8e-04$       | 9.439e+05    | $\pm 1.4e+04$       | 0.99982    | 2.594e-06                |
| EUR-GBP          | 0.450        | $\pm 4.0e-04$       | 2.806e+05    | $\pm 4.0e+03$       | 0.99980    | 3.721e-06                |
| EUR-JPY          | 0.481        | $\pm 6.4e-04$       | 1.229e+05    | $\pm 2.5e+03$       | 0.99957    | 2.387e-04                |
| EUR-USD          | 0.494        | $\pm 1.1e-03$       | 1.337e+05    | $\pm 4.4e+03$       | 0.99887    | 8.416e-04                |
| GBP-CHF          | 0.456        | $\pm 5.6e-04$       | 2.050e+05    | $\pm 3.9e+03$       | 0.99963    | 1.333e-04                |
| GBP-JPY          | 0.475        | $\pm 7.4e-04$       | 1.106e+05    | $\pm 2.6e+03$       | 0.99940    | 2.845e-04                |
| GBP-USD          | 0.495        | $\pm 9.7e-04$       | 1.529e+05    | $\pm 4.6e+03$       | 0.99907    | 6.398e-04                |
| GRW              | 0.510        | $\pm 1.0e-03$       | 2.739e+03    | $\pm 6.5e+01$       | 0.99914    | 1.415e-04                |
| USD-CHF          | 0.493        | $\pm 1.1e-03$       | 1.080e+05    | $\pm 3.8e+03$       | 0.99871    | 1.042e-03                |
| USD-JPY          | 0.485        | $\pm 1.0e-03$       | 1.259e+05    | $\pm 4.1e+03$       | 0.99888    | 8.555e-04                |
| Currency average | 0.4728       | (1.8e-0)            | 1.98e+05     | (2.3e+05)           |            |                          |

Table A8: Time of price move, law (4)

| Currency         | $E_{t,x}$ | $\Delta E_{t,x}$ | $C_{t,x}$ | $\Delta C_{t,x}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|-----------|------------------|-----------|------------------|------------|--------------------------|
| AUD-JPY          | 2.051     | $\pm 4.0e-03$    | 2.477e-03 | $\pm 3.1e-05$    | 0.99906    | 4.705e-05                |
| AUD-USD          | 1.972     | $\pm 3.8e-03$    | 1.774e-03 | $\pm 2.3e-05$    | 0.99907    | -3.584e-06               |
| CHF-JPY          | 2.088     | $\pm 5.4e-03$    | 2.109e-03 | $\pm 3.6e-05$    | 0.99835    | 1.145e-04                |
| EUR-AUD          | 2.134     | $\pm 4.2e-03$    | 2.451e-03 | $\pm 3.1e-05$    | 0.99905    | 1.803e-04                |
| EUR-CHF          | 2.125     | $\pm 5.1e-03$    | 9.598e-04 | $\pm 1.7e-05$    | 0.99858    | 6.277e-04                |
| EUR-GBP          | 2.184     | $\pm 9.4e-03$    | 1.905e-03 | $\pm 5.5e-05$    | 0.99538    | 1.725e-03                |
| EUR-JPY          | 1.999     | $\pm 2.9e-03$    | 1.627e-03 | $\pm 1.6e-05$    | 0.99947    | 1.794e-04                |
| EUR-USD          | 1.928     | $\pm 3.2e-03$    | 1.227e-03 | $\pm 1.4e-05$    | 0.99933    | 1.853e-04                |
| GBP-CHF          | 2.123     | $\pm 3.0e-03$    | 1.825e-03 | $\pm 1.7e-05$    | 0.99951    | 1.237e-04                |
| GBP-JPY          | 2.028     | $\pm 2.7e-03$    | 1.841e-03 | $\pm 1.7e-05$    | 0.99954    | -1.336e-06               |
| GBP-USD          | 1.932     | $\pm 2.2e-03$    | 1.162e-03 | $\pm 9.5e-06$    | 0.99967    | 6.721e-05                |
| GRW              | 1.864     | $\pm 6.0e-03$    | 8.640e-03 | $\pm 1.5e-04$    | 0.99740    | 1.843e-03                |
| USD-CHF          | 1.945     | $\pm 3.2e-03$    | 1.428e-03 | $\pm 1.6e-05$    | 0.99932    | 2.213e-04                |
| USD-JPY          | 1.977     | $\pm 4.0e-03$    | 1.461e-03 | $\pm 2.0e-05$    | 0.99898    | 4.814e-04                |
| Currency average | 2.04      | (8.6e-02)        | 1.71e-03  | (4.7e-04)        |            |                          |

Table A9: Time between directional changes, law (5)

| Currency         | $E_{t,dc}$ | $\Delta E_{t,dc}$ | $C_{t,dc}$ | $\Delta C_{t,dc}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | 2.046      | $\pm 4.5e-03$     | 2.439e-03  | $\pm 3.5e-05$     | 0.99877    | 2.749e-04                |
| AUD-USD          | 1.948      | $\pm 4.4e-03$     | 1.608e-03  | $\pm 2.5e-05$     | 0.99873    | 4.641e-04                |
| CHF-JPY          | 2.063      | $\pm 4.8e-03$     | 2.053e-03  | $\pm 3.2e-05$     | 0.99863    | 1.438e-04                |
| EUR-AUD          | 2.132      | $\pm 5.8e-03$     | 2.531e-03  | $\pm 4.4e-05$     | 0.99817    | 6.042e-05                |
| EUR-CHF          | 2.111      | $\pm 6.8e-03$     | 9.819e-04  | $\pm 2.3e-05$     | 0.99741    | 1.586e-03                |
| EUR-GBP          | 2.162      | $\pm 8.3e-03$     | 1.907e-03  | $\pm 4.9e-05$     | 0.99635    | 1.536e-03                |
| EUR-JPY          | 2.001      | $\pm 2.9e-03$     | 1.629e-03  | $\pm 1.6e-05$     | 0.99947    | 1.180e-04                |
| EUR-USD          | 1.884      | $\pm 3.9e-03$     | 1.050e-03  | $\pm 1.6e-05$     | 0.99893    | 3.450e-04                |
| GBP-CHF          | 2.127      | $\pm 3.2e-03$     | 1.926e-03  | $\pm 1.9e-05$     | 0.99944    | 7.375e-05                |
| GBP-JPY          | 2.016      | $\pm 3.4e-03$     | 1.804e-03  | $\pm 2.0e-05$     | 0.99930    | -2.835e-06               |
| GBP-USD          | 1.899      | $\pm 3.2e-03$     | 1.020e-03  | $\pm 1.3e-05$     | 0.99928    | 1.643e-04                |
| GRW              | 1.790      | $\pm 9.1e-03$     | 6.953e-03  | $\pm 1.9e-04$     | 0.99361    | 4.807e-03                |
| USD-CHF          | 1.904      | $\pm 3.1e-03$     | 1.247e-03  | $\pm 1.5e-05$     | 0.99932    | 2.326e-04                |
| USD-JPY          | 1.927      | $\pm 4.6e-03$     | 1.266e-03  | $\pm 2.1e-05$     | 0.99857    | 7.180e-04                |
| Currency average | 2.02       | (9.8e-02)         | 1.65e-03   | (5.2e-04)         |            |                          |

Table A10: Total price move, law (9), \* =  $tm$ 

| Currency         | $E_{x,tm}$ | $\Delta E_{x,tm}$ | $C_{x,tm}$ | $\Delta C_{x,tm}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | 1.001      | $\pm 2.3e-03$     | 4.998e-01  | $\pm 2.8e-03$     | 0.99872    | 4.740e-04                |
| AUD-USD          | 0.990      | $\pm 2.0e-03$     | 4.876e-01  | $\pm 2.4e-03$     | 0.99899    | 2.066e-04                |
| CHF-JPY          | 0.995      | $\pm 2.3e-03$     | 5.211e-01  | $\pm 2.9e-03$     | 0.99868    | 2.848e-05                |
| EUR-AUD          | 0.997      | $\pm 2.4e-03$     | 5.232e-01  | $\pm 3.1e-03$     | 0.99851    | 3.814e-04                |
| EUR-CHF          | 1.006      | $\pm 2.3e-03$     | 5.299e-01  | $\pm 3.0e-03$     | 0.99865    | 2.258e-04                |
| EUR-GBP          | 1.004      | $\pm 3.0e-03$     | 5.388e-01  | $\pm 3.9e-03$     | 0.99782    | 2.652e-04                |
| EUR-JPY          | 1.001      | $\pm 1.3e-03$     | 5.001e-01  | $\pm 1.6e-03$     | 0.99956    | 6.695e-06                |
| EUR-USD          | 0.976      | $\pm 1.7e-03$     | 4.871e-01  | $\pm 2.1e-03$     | 0.99921    | 2.324e-04                |
| GBP-CHF          | 0.993      | $\pm 1.6e-03$     | 5.358e-01  | $\pm 2.2e-03$     | 0.99932    | 4.097e-05                |
| GBP-JPY          | 0.996      | $\pm 1.5e-03$     | 5.039e-01  | $\pm 1.9e-03$     | 0.99940    | 2.969e-05                |
| GBP-USD          | 0.981      | $\pm 1.2e-03$     | 4.885e-01  | $\pm 1.5e-03$     | 0.99963    | 3.725e-05                |
| GRW              | 0.943      | $\pm 3.4e-03$     | 4.708e-01  | $\pm 4.2e-03$     | 0.99670    | 2.060e-03                |
| USD-CHF          | 0.973      | $\pm 1.4e-03$     | 4.984e-01  | $\pm 1.7e-03$     | 0.99950    | -1.570e-06               |
| USD-JPY          | 0.969      | $\pm 1.8e-03$     | 5.028e-01  | $\pm 2.3e-03$     | 0.99913    | 1.081e-04                |
| Currency average | 0.99       | (1.2e-02)         | 5.10e-01   | (1.9e-02)         |            |                          |

Table A11: Directional-change move, law (9), \* =  $dc$ 

| Currency         | $E_{x,dc}$ | $\Delta E_{x,dc}$ | $C_{x,dc}$ | $\Delta C_{x,dc}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | 0.941      | $\pm 2.5e-03$     | 9.847e-01  | $\pm 6.1e-03$     | 0.99828    | 1.033e-03                |
| AUD-USD          | 0.937      | $\pm 2.4e-03$     | 9.814e-01  | $\pm 5.9e-03$     | 0.99838    | 9.673e-04                |
| CHF-JPY          | 0.949      | $\pm 2.6e-03$     | 9.898e-01  | $\pm 6.3e-03$     | 0.99816    | 8.473e-04                |
| EUR-AUD          | 0.946      | $\pm 2.0e-03$     | 9.843e-01  | $\pm 5.0e-03$     | 0.99884    | 8.851e-04                |
| EUR-CHF          | 0.964      | $\pm 1.5e-03$     | 9.907e-01  | $\pm 3.7e-03$     | 0.99937    | 3.488e-04                |
| EUR-GBP          | 0.948      | $\pm 2.6e-03$     | 9.827e-01  | $\pm 6.2e-03$     | 0.99818    | 5.941e-04                |
| EUR-JPY          | 0.961      | $\pm 1.7e-03$     | 9.921e-01  | $\pm 4.0e-03$     | 0.99926    | 4.974e-04                |
| EUR-USD          | 0.959      | $\pm 1.9e-03$     | 9.918e-01  | $\pm 4.6e-03$     | 0.99902    | 6.268e-04                |
| GBP-CHF          | 0.961      | $\pm 1.6e-03$     | 9.918e-01  | $\pm 3.8e-03$     | 0.99934    | 4.941e-04                |
| GBP-JPY          | 0.959      | $\pm 1.7e-03$     | 9.926e-01  | $\pm 4.1e-03$     | 0.99922    | 5.991e-04                |
| GBP-USD          | 0.961      | $\pm 1.5e-03$     | 9.918e-01  | $\pm 3.6e-03$     | 0.99940    | 4.778e-04                |
| GRW              | 0.937      | $\pm 2.6e-03$     | 9.943e-01  | $\pm 6.5e-03$     | 0.99805    | 1.656e-03                |
| USD-CHF          | 0.958      | $\pm 1.9e-03$     | 9.913e-01  | $\pm 4.6e-03$     | 0.99901    | 6.267e-04                |
| USD-JPY          | 0.954      | $\pm 2.3e-03$     | 9.934e-01  | $\pm 5.6e-03$     | 0.99858    | 8.996e-04                |
| Currency average | 0.95       | (8.6e-03)         | 9.89e-01   | (4.2e-03)         |            |                          |

Table A12: Overshoot move, law (9), \* =  $os$ 

| Currency         | $E_{x,os}$ | $\Delta E_{x,os}$ | $C_{x,os}$ | $\Delta C_{x,os}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | 1.081      | $\pm 2.5e-03$     | 1.012e+00  | $\pm 5.5e-03$     | 0.99865    | -4.901e-06               |
| AUD-USD          | 1.060      | $\pm 2.8e-03$     | 9.788e-01  | $\pm 6.1e-03$     | 0.99823    | 1.273e-04                |
| CHF-JPY          | 1.064      | $\pm 4.9e-03$     | 1.098e+00  | $\pm 1.2e-02$     | 0.99478    | 3.123e-03                |
| EUR-AUD          | 1.066      | $\pm 3.8e-03$     | 1.121e+00  | $\pm 9.3e-03$     | 0.99692    | -9.706e-06               |
| EUR-CHF          | 1.071      | $\pm 6.0e-03$     | 1.118e+00  | $\pm 1.5e-02$     | 0.99228    | 3.963e-03                |
| EUR-GBP          | 1.103      | $\pm 7.4e-03$     | 1.164e+00  | $\pm 1.8e-02$     | 0.98905    | 7.059e-03                |
| EUR-JPY          | 1.052      | $\pm 2.9e-03$     | 1.004e+00  | $\pm 6.4e-03$     | 0.99815    | 1.110e-03                |
| EUR-USD          | 0.996      | $\pm 2.2e-03$     | 9.879e-01  | $\pm 5.1e-03$     | 0.99880    | 1.930e-05                |
| GBP-CHF          | 1.041      | $\pm 4.0e-03$     | 1.168e+00  | $\pm 1.1e-02$     | 0.99629    | 2.002e-03                |
| GBP-JPY          | 1.042      | $\pm 2.7e-03$     | 1.027e+00  | $\pm 6.3e-03$     | 0.99830    | 3.072e-04                |
| GBP-USD          | 1.003      | $\pm 2.0e-03$     | 9.871e-01  | $\pm 4.6e-03$     | 0.99900    | 1.002e-04                |
| GRW              | 0.945      | $\pm 4.7e-03$     | 9.812e-01  | $\pm 1.1e-02$     | 0.99390    | 2.167e-03                |
| USD-CHF          | 0.990      | $\pm 2.8e-03$     | 1.045e+00  | $\pm 7.0e-03$     | 0.99798    | 9.718e-04                |
| USD-JPY          | 0.987      | $\pm 2.6e-03$     | 1.067e+00  | $\pm 6.6e-03$     | 0.99825    | 1.625e-04                |
| Currency average | 1.04       | (3.8e-02)         | 1.06e+00   | (6.8e-02)         |            |                          |

Table A13: Time of total move, law (10), \* =  $tm$ 

| Currency         | $E_{t,tm}$ | $\Delta E_{t,tm}$ | $C_{t,tm}$ | $\Delta C_{t,tm}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | 2.049      | $\pm 4.6e-03$     | 2.450e-03  | $\pm 3.5e-05$     | 0.99876    | 3.127e-04                |
| AUD-USD          | 1.950      | $\pm 4.5e-03$     | 1.616e-03  | $\pm 2.5e-05$     | 0.99870    | 5.183e-04                |
| CHF-JPY          | 2.066      | $\pm 4.8e-03$     | 2.063e-03  | $\pm 3.1e-05$     | 0.99868    | 1.143e-04                |
| EUR-AUD          | 2.133      | $\pm 5.8e-03$     | 2.534e-03  | $\pm 4.4e-05$     | 0.99816    | 6.282e-05                |
| EUR-CHF          | 2.103      | $\pm 8.5e-03$     | 9.655e-04  | $\pm 2.9e-05$     | 0.99594    | 2.089e-03                |
| EUR-GBP          | 2.167      | $\pm 8.1e-03$     | 1.926e-03  | $\pm 4.8e-05$     | 0.99653    | 1.311e-03                |
| EUR-JPY          | 2.004      | $\pm 2.8e-03$     | 1.638e-03  | $\pm 1.6e-05$     | 0.99952    | 8.980e-05                |
| EUR-USD          | 1.886      | $\pm 3.9e-03$     | 1.053e-03  | $\pm 1.6e-05$     | 0.99892    | 3.705e-04                |
| GBP-CHF          | 2.124      | $\pm 3.3e-03$     | 1.919e-03  | $\pm 2.0e-05$     | 0.99941    | 8.651e-05                |
| GBP-JPY          | 2.018      | $\pm 3.4e-03$     | 1.809e-03  | $\pm 2.0e-05$     | 0.99931    | -2.651e-06               |
| GBP-USD          | 1.902      | $\pm 3.2e-03$     | 1.026e-03  | $\pm 1.3e-05$     | 0.99928    | 1.987e-04                |
| GRW              | 1.791      | $\pm 9.3e-03$     | 6.967e-03  | $\pm 2.0e-04$     | 0.99338    | 4.870e-03                |
| USD-CHF          | 1.898      | $\pm 3.1e-03$     | 1.229e-03  | $\pm 1.4e-05$     | 0.99934    | 1.430e-04                |
| USD-JPY          | 1.925      | $\pm 4.6e-03$     | 1.260e-03  | $\pm 2.1e-05$     | 0.99857    | 6.307e-04                |
| Currency average | 2.02       | (9.8e-02)         | 1.65e-03   | (5.3e-04)         |            |                          |

Table A14: Time of directional change, law (10), \* =  $dc$ 

| Currency         | $E_{t,dc}$ | $\Delta E_{t,dc}$ | $C_{t,dc}$ | $\Delta C_{t,dc}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | 1.996      | $\pm 4.5e-03$     | 3.654e-03  | $\pm 5.0e-05$     | 0.99875    | 2.448e-04                |
| AUD-USD          | 1.871      | $\pm 4.1e-03$     | 2.280e-03  | $\pm 3.2e-05$     | 0.99882    | 4.129e-04                |
| CHF-JPY          | 2.015      | $\pm 5.5e-03$     | 3.096e-03  | $\pm 5.3e-05$     | 0.99812    | 5.073e-04                |
| EUR-AUD          | 2.102      | $\pm 4.7e-03$     | 3.789e-03  | $\pm 5.1e-05$     | 0.99875    | 1.251e-04                |
| EUR-CHF          | 2.066      | $\pm 7.1e-03$     | 1.463e-03  | $\pm 3.5e-05$     | 0.99709    | 2.023e-03                |
| EUR-GBP          | 2.135      | $\pm 6.3e-03$     | 2.838e-03  | $\pm 5.3e-05$     | 0.99786    | 4.578e-04                |
| EUR-JPY          | 1.941      | $\pm 4.1e-03$     | 2.415e-03  | $\pm 3.3e-05$     | 0.99887    | 6.149e-04                |
| EUR-USD          | 1.846      | $\pm 3.4e-03$     | 1.636e-03  | $\pm 2.1e-05$     | 0.99915    | 3.813e-05                |
| GBP-CHF          | 2.099      | $\pm 3.9e-03$     | 2.874e-03  | $\pm 3.4e-05$     | 0.99914    | 1.035e-04                |
| GBP-JPY          | 1.958      | $\pm 3.1e-03$     | 2.626e-03  | $\pm 2.7e-05$     | 0.99938    | 1.054e-04                |
| GBP-USD          | 1.866      | $\pm 2.4e-03$     | 1.596e-03  | $\pm 1.4e-05$     | 0.99960    | 6.737e-05                |
| GRW              | 1.774      | $\pm 9.8e-03$     | 1.235e-02  | $\pm 3.4e-04$     | 0.99240    | 6.439e-03                |
| USD-CHF          | 1.888      | $\pm 2.8e-03$     | 2.016e-03  | $\pm 2.0e-05$     | 0.99945    | 5.768e-05                |
| USD-JPY          | 1.914      | $\pm 4.1e-03$     | 2.057e-03  | $\pm 2.9e-05$     | 0.99885    | 5.935e-04                |
| Currency average | 1.98       | (1.0e-01)         | 2.49e-03   | (7.5e-04)         |            |                          |

Table A15: Time of overshoot, law (10), \* = *os*

| Currency         | $E_{t,os}$ | $\Delta E_{t,os}$ | $C_{t,os}$ | $\Delta C_{t,os}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | 2.079      | $\pm 5.4e-03$     | 3.241e-03  | $\pm 5.2e-05$     | 0.99831    | 2.681e-04                |
| AUD-USD          | 1.997      | $\pm 5.5e-03$     | 2.273e-03  | $\pm 4.1e-05$     | 0.99810    | 3.617e-04                |
| CHF-JPY          | 2.093      | $\pm 5.8e-03$     | 2.698e-03  | $\pm 4.7e-05$     | 0.99812    | 4.505e-05                |
| EUR-AUD          | 2.150      | $\pm 6.8e-03$     | 3.274e-03  | $\pm 6.4e-05$     | 0.99750    | 2.799e-05                |
| EUR-CHF          | 2.119      | $\pm 1.0e-02$     | 1.234e-03  | $\pm 4.2e-05$     | 0.99441    | 2.222e-03                |
| EUR-GBP          | 2.190      | $\pm 1.1e-02$     | 2.519e-03  | $\pm 7.8e-05$     | 0.99420    | 2.342e-03                |
| EUR-JPY          | 2.035      | $\pm 3.1e-03$     | 2.182e-03  | $\pm 2.1e-05$     | 0.99944    | 4.337e-06                |
| EUR-USD          | 1.906      | $\pm 5.2e-03$     | 1.411e-03  | $\pm 2.7e-05$     | 0.99816    | 6.454e-04                |
| GBP-CHF          | 2.136      | $\pm 4.9e-03$     | 2.469e-03  | $\pm 3.7e-05$     | 0.99869    | 1.094e-04                |
| GBP-JPY          | 2.049      | $\pm 4.4e-03$     | 2.435e-03  | $\pm 3.4e-05$     | 0.99884    | 2.919e-05                |
| GBP-USD          | 1.921      | $\pm 4.4e-03$     | 1.372e-03  | $\pm 2.2e-05$     | 0.99871    | 2.923e-04                |
| GRW              | 1.796      | $\pm 9.6e-03$     | 8.900e-03  | $\pm 2.5e-04$     | 0.99295    | 3.823e-03                |
| USD-CHF          | 1.903      | $\pm 4.0e-03$     | 1.597e-03  | $\pm 2.3e-05$     | 0.99891    | 2.088e-04                |
| USD-JPY          | 1.930      | $\pm 5.5e-03$     | 1.624e-03  | $\pm 3.2e-05$     | 0.99796    | 6.314e-04                |
| Currency average | 2.04       | (1.0e-01)         | 2.18e-03   | (6.9e-04)         |            |                          |

Table A16: Total-move tick count, law (11), \* = *tm*

| Currency         | $E_{N,tm}$ | $\Delta E_{N,tm}$ | $C_{N,tm}$ | $\Delta C_{N,tm}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | 2.048      | $\pm 4.6e-03$     | 1.861e-02  | $\pm 1.9e-04$     | 0.99877    | 3.049e-04                |
| AUD-USD          | 1.949      | $\pm 4.4e-03$     | 2.008e-02  | $\pm 2.1e-04$     | 0.99873    | 4.784e-04                |
| CHF-JPY          | 2.063      | $\pm 4.9e-03$     | 1.661e-02  | $\pm 1.8e-04$     | 0.99862    | 1.473e-04                |
| EUR-AUD          | 2.131      | $\pm 5.8e-03$     | 2.098e-02  | $\pm 2.6e-04$     | 0.99813    | 5.135e-05                |
| EUR-CHF          | 2.121      | $\pm 6.3e-03$     | 2.166e-02  | $\pm 2.9e-04$     | 0.99778    | 1.058e-03                |
| EUR-GBP          | 2.177      | $\pm 7.7e-03$     | 2.225e-02  | $\pm 3.5e-04$     | 0.99687    | 9.997e-04                |
| EUR-JPY          | 2.003      | $\pm 2.8e-03$     | 1.864e-02  | $\pm 1.2e-04$     | 0.99950    | 1.014e-04                |
| EUR-USD          | 1.893      | $\pm 4.1e-03$     | 1.931e-02  | $\pm 1.9e-04$     | 0.99884    | 5.552e-04                |
| GBP-CHF          | 2.121      | $\pm 3.4e-03$     | 2.024e-02  | $\pm 1.5e-04$     | 0.99937    | 1.176e-04                |
| GBP-JPY          | 2.016      | $\pm 3.4e-03$     | 1.909e-02  | $\pm 1.5e-04$     | 0.99927    | -2.765e-06               |
| GBP-USD          | 1.902      | $\pm 3.2e-03$     | 1.883e-02  | $\pm 1.4e-04$     | 0.99931    | 2.122e-04                |
| GRW              | 1.792      | $\pm 9.3e-03$     | 1.767e-02  | $\pm 4.3e-04$     | 0.99338    | 4.899e-03                |
| USD-CHF          | 1.898      | $\pm 3.0e-03$     | 1.866e-02  | $\pm 1.4e-04$     | 0.99937    | 1.604e-04                |
| USD-JPY          | 1.922      | $\pm 4.6e-03$     | 2.047e-02  | $\pm 2.2e-04$     | 0.99858    | 5.640e-04                |
| Currency average | 2.02       | (1.0e-02)         | 1.97e-02   | (1.5e-03)         |            |                          |

Table A17: Directional-change tick count, law (11), \* =  $dc$ 

| Currency         | $E_{N,dc}$ | $\Delta E_{N,dc}$ | $C_{N,dc}$ | $\Delta C_{N,dc}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | 2.002      | $\pm 7.5e-03$     | 2.974e-02  | $\pm 4.7e-04$     | 0.99647    | 7.871e-04                |
| AUD-USD          | 1.904      | $\pm 7.8e-03$     | 3.375e-02  | $\pm 5.7e-04$     | 0.99583    | 2.323e-04                |
| CHF-JPY          | 2.009      | $\pm 8.1e-03$     | 2.652e-02  | $\pm 4.6e-04$     | 0.99600    | -4.305e-06               |
| EUR-AUD          | 2.128      | $\pm 4.6e-03$     | 3.406e-02  | $\pm 3.0e-04$     | 0.99884    | 9.624e-06                |
| EUR-CHF          | 2.198      | $\pm 1.0e-02$     | 3.896e-02  | $\pm 7.4e-04$     | 0.99442    | 2.203e-03                |
| EUR-GBP          | 2.205      | $\pm 1.5e-02$     | 3.748e-02  | $\pm 1.0e-03$     | 0.98832    | 1.535e-03                |
| EUR-JPY          | 2.040      | $\pm 9.4e-03$     | 3.465e-02  | $\pm 6.6e-04$     | 0.99471    | 1.755e-03                |
| EUR-USD          | 2.017      | $\pm 1.2e-02$     | 4.187e-02  | $\pm 1.0e-03$     | 0.99088    | 2.739e-03                |
| GBP-CHF          | 2.181      | $\pm 6.4e-03$     | 3.548e-02  | $\pm 4.2e-04$     | 0.99786    | 7.460e-04                |
| GBP-JPY          | 2.048      | $\pm 5.2e-03$     | 3.436e-02  | $\pm 3.6e-04$     | 0.99839    | 5.759e-04                |
| GBP-USD          | 1.985      | $\pm 7.0e-03$     | 3.782e-02  | $\pm 5.3e-04$     | 0.99694    | 9.365e-04                |
| GRW              | 1.809      | $\pm 6.6e-03$     | 3.350e-02  | $\pm 5.0e-04$     | 0.99669    | 2.568e-03                |
| USD-CHF          | 1.991      | $\pm 9.4e-03$     | 3.772e-02  | $\pm 7.2e-04$     | 0.99451    | 7.606e-04                |
| USD-JPY          | 2.052      | $\pm 9.9e-03$     | 4.266e-02  | $\pm 8.1e-04$     | 0.99427    | 9.073e-04                |
| Currency average | 2.06       | (9.2e-02)         | 3.58e-02   | (4.5e-03)         |            |                          |

Table A18: Overshoot tick count, law (11), \* =  $os$ 

| Currency         | $E_{N,os}$ | $\Delta E_{N,os}$ | $C_{N,os}$ | $\Delta C_{N,os}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|------------|-------------------|------------|-------------------|------------|--------------------------|
| AUD-JPY          | 2.081      | $\pm 4.7e-03$     | 2.395e-02  | $\pm 2.4e-04$     | 0.99873    | 5.969e-05                |
| AUD-USD          | 1.978      | $\pm 4.5e-03$     | 2.573e-02  | $\pm 2.5e-04$     | 0.99873    | 4.303e-04                |
| CHF-JPY          | 2.101      | $\pm 6.0e-03$     | 2.140e-02  | $\pm 2.7e-04$     | 0.99799    | 5.063e-04                |
| EUR-AUD          | 2.137      | $\pm 7.9e-03$     | 2.602e-02  | $\pm 4.2e-04$     | 0.99658    | 3.846e-05                |
| EUR-CHF          | 2.089      | $\pm 7.9e-03$     | 2.554e-02  | $\pm 4.2e-04$     | 0.99642    | 1.126e-03                |
| EUR-GBP          | 2.182      | $\pm 8.9e-03$     | 2.743e-02  | $\pm 4.8e-04$     | 0.99587    | 1.462e-03                |
| EUR-JPY          | 1.998      | $\pm 4.0e-03$     | 2.246e-02  | $\pm 2.0e-04$     | 0.99902    | -8.981e-07               |
| EUR-USD          | 1.868      | $\pm 6.6e-03$     | 2.277e-02  | $\pm 3.6e-04$     | 0.99687    | 1.731e-03                |
| GBP-CHF          | 2.095      | $\pm 6.0e-03$     | 2.416e-02  | $\pm 3.0e-04$     | 0.99796    | 7.889e-05                |
| GBP-JPY          | 2.006      | $\pm 5.3e-03$     | 2.306e-02  | $\pm 2.7e-04$     | 0.99828    | 4.535e-05                |
| GBP-USD          | 1.878      | $\pm 5.2e-03$     | 2.239e-02  | $\pm 2.8e-04$     | 0.99806    | 7.896e-04                |
| GRW              | 1.783      | $\pm 1.1e-02$     | 2.194e-02  | $\pm 6.2e-04$     | 0.99034    | 5.591e-03                |
| USD-CHF          | 1.871      | $\pm 4.5e-03$     | 2.220e-02  | $\pm 2.4e-04$     | 0.99855    | 4.614e-04                |
| USD-JPY          | 1.884      | $\pm 6.8e-03$     | 2.405e-02  | $\pm 3.8e-04$     | 0.99678    | 1.210e-03                |
| Currency average | 2.01       | (1.1e-01)         | 2.39e-02   | (1.8e-03)         |            |                          |

Table A19: Cumulative total move (coastline), law (12), \* =  $tm$ 

| Currency         | $E_{cum,tm}$ | $\Delta E_{cum,tm}$ | $C_{cum,tm}$ | $\Delta C_{cum,tm}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|--------------|---------------------|--------------|---------------------|------------|--------------------------|
| AUD-JPY          | -1.048       | $\pm 2.6e-03$       | 2.139e+02    | $\pm 3.1e+00$       | 0.99850    | 1.949e-04                |
| AUD-USD          | -0.961       | $\pm 2.7e-03$       | 2.861e+02    | $\pm 4.9e+00$       | 0.99805    | 1.054e-03                |
| CHF-JPY          | -1.076       | $\pm 2.5e-03$       | 1.154e+02    | $\pm 1.4e+00$       | 0.99868    | 1.307e-04                |
| EUR-AUD          | -1.140       | $\pm 3.7e-03$       | 9.003e+01    | $\pm 1.5e+00$       | 0.99738    | -7.231e-06               |
| EUR-CHF          | -1.189       | $\pm 5.5e-03$       | 1.312e+01    | $\pm 2.1e-01$       | 0.99461    | 1.357e-04                |
| EUR-GBP          | -1.184       | $\pm 4.9e-03$       | 3.711e+01    | $\pm 6.6e-01$       | 0.99579    | 1.347e-03                |
| EUR-JPY          | -1.005       | $\pm 1.6e-03$       | 1.601e+02    | $\pm 1.4e+00$       | 0.99938    | 2.140e-04                |
| EUR-USD          | -0.937       | $\pm 3.8e-03$       | 2.009e+02    | $\pm 4.8e+00$       | 0.99583    | 2.997e-03                |
| GBP-CHF          | -1.145       | $\pm 2.1e-03$       | 5.351e+01    | $\pm 4.5e-01$       | 0.99916    | -3.260e-06               |
| GBP-JPY          | -1.024       | $\pm 1.9e-03$       | 1.605e+02    | $\pm 1.7e+00$       | 0.99910    | 4.353e-07                |
| GBP-USD          | -0.929       | $\pm 2.4e-03$       | 1.878e+02    | $\pm 2.8e+00$       | 0.99832    | 9.695e-04                |
| GRW              | -0.868       | $\pm 6.8e-03$       | 6.157e+02    | $\pm 3.3e+01$       | 0.98509    | 1.284e-02                |
| USD-CHF          | -0.939       | $\pm 2.7e-03$       | 2.515e+02    | $\pm 4.3e+00$       | 0.99796    | 1.580e-03                |
| USD-JPY          | -0.963       | $\pm 3.3e-03$       | 1.921e+02    | $\pm 3.8e+00$       | 0.99701    | 2.282e-03                |
| Currency average | -1.04        | (9.7e-02)           | 1.51e+02     | (8.4e+01)           |            |                          |

Table A20: Cumulative cost-adjusted total move, law (12), \* =  $tm$ , fitted from 0.2%

| Currency         | $E_{cum,tm}$ | $\Delta E_{cum,tm}$ | $C_{cum,tm}$ | $\Delta C_{cum,tm}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|--------------|---------------------|--------------|---------------------|------------|--------------------------|
| AUD-JPY          | -0.941       | $\pm 2.9e-03$       | 3.378e+02    | $\pm 6.2e+00$       | 0.99874    | 3.358e-05                |
| AUD-USD          | -0.943       | $\pm 2.0e-03$       | 2.920e+02    | $\pm 3.6e+00$       | 0.99940    | 7.101e-05                |
| CHF-JPY          | -0.956       | $\pm 3.4e-03$       | 1.854e+02    | $\pm 3.5e+00$       | 0.99838    | 1.087e-03                |
| EUR-AUD          | -0.970       | $\pm 4.9e-03$       | 1.652e+02    | $\pm 4.3e+00$       | 0.99678    | 1.822e-03                |
| EUR-CHF          | -1.201       | $\pm 2.0e-02$       | 1.254e+01    | $\pm 5.5e-01$       | 0.96693    | 1.789e-02                |
| EUR-GBP          | -1.034       | $\pm 7.9e-03$       | 5.743e+01    | $\pm 1.8e+00$       | 0.99249    | 3.734e-03                |
| EUR-JPY          | -0.915       | $\pm 2.0e-03$       | 2.454e+02    | $\pm 2.9e+00$       | 0.99941    | 1.810e-04                |
| EUR-USD          | -0.980       | $\pm 8.2e-03$       | 1.524e+02    | $\pm 6.5e+00$       | 0.99102    | 5.138e-03                |
| GBP-CHF          | -1.059       | $\pm 7.4e-03$       | 6.783e+01    | $\pm 2.1e+00$       | 0.99366    | 3.061e-03                |
| GBP-JPY          | -0.912       | $\pm 2.3e-03$       | 2.666e+02    | $\pm 3.8e+00$       | 0.99919    | 9.254e-06                |
| GBP-USD          | -0.916       | $\pm 4.2e-03$       | 1.903e+02    | $\pm 4.6e+00$       | 0.99730    | 7.322e-04                |
| GRW              | -0.994       | $\pm 9.2e-03$       | 2.588e+02    | $\pm 1.4e+01$       | 0.98897    | 3.848e-03                |
| USD-CHF          | -0.949       | $\pm 5.4e-03$       | 2.214e+02    | $\pm 6.9e+00$       | 0.99582    | 3.302e-03                |
| USD-JPY          | -0.979       | $\pm 4.4e-03$       | 1.650e+02    | $\pm 3.8e+00$       | 0.99744    | 1.522e-03                |
| Currency average | -0.98        | (7.9e-02)           | 1.82e+02     | (9.5e+01)           |            |                          |

(For the GRW a constant spread of 0.02% was introduced.)

Table A21: Cumulative directional change, law (12), \* =  $dc$ 

| Currency         | $E_{cum,dc}$ | $\Delta E_{cum,dc}$ | $C_{cum,dc}$ | $\Delta C_{cum,dc}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|--------------|---------------------|--------------|---------------------|------------|--------------------------|
| AUD-JPY          | -1.108       | $\pm 2.8e-03$       | 8.654e+01    | $\pm 1.1e+00$       | 0.99843    | 2.727e-05                |
| AUD-USD          | -1.014       | $\pm 2.3e-03$       | 1.075e+02    | $\pm 1.3e+00$       | 0.99867    | 2.578e-04                |
| CHF-JPY          | -1.121       | $\pm 4.0e-03$       | 5.387e+01    | $\pm 9.0e-01$       | 0.99678    | 1.674e-03                |
| EUR-AUD          | -1.191       | $\pm 4.8e-03$       | 4.376e+01    | $\pm 7.9e-01$       | 0.99590    | 1.308e-05                |
| EUR-CHF          | -1.232       | $\pm 6.2e-03$       | 7.196e+00    | $\pm 1.1e-01$       | 0.99367    | 1.967e-04                |
| EUR-GBP          | -1.240       | $\pm 7.0e-03$       | 1.940e+01    | $\pm 4.1e-01$       | 0.99213    | 4.493e-03                |
| EUR-JPY          | -1.044       | $\pm 3.0e-03$       | 6.849e+01    | $\pm 9.5e-01$       | 0.99797    | 1.399e-03                |
| EUR-USD          | -0.954       | $\pm 3.8e-03$       | 8.805e+01    | $\pm 1.8e+00$       | 0.99597    | 1.953e-03                |
| GBP-CHF          | -1.178       | $\pm 3.1e-03$       | 2.851e+01    | $\pm 3.0e-01$       | 0.99831    | 5.737e-04                |
| GBP-JPY          | -1.061       | $\pm 2.9e-03$       | 7.116e+01    | $\pm 9.5e-01$       | 0.99813    | 3.460e-04                |
| GBP-USD          | -0.948       | $\pm 2.2e-03$       | 8.117e+01    | $\pm 9.2e-01$       | 0.99872    | 2.150e-04                |
| GRW              | -0.874       | $\pm 7.6e-03$       | 2.630e+02    | $\pm 1.4e+01$       | 0.98147    | 1.385e-02                |
| USD-CHF          | -0.954       | $\pm 1.9e-03$       | 1.143e+02    | $\pm 1.2e+00$       | 0.99901    | 1.745e-04                |
| USD-JPY          | -0.978       | $\pm 2.7e-03$       | 9.014e+01    | $\pm 1.3e+00$       | 0.99811    | 7.933e-04                |
| Currency average | -1.08        | (1.1e-01)           | 6.62e+01     | (3.4e+01)           |            |                          |

Table A22: Cumulative overshoot, law (12), \* =  $os$ 

| Currency         | $E_{cum,os}$ | $\Delta E_{cum,os}$ | $C_{cum,os}$ | $\Delta C_{cum,os}$ | Adj. $R^2$ | $R_{quad}^2 - R_{lin}^2$ |
|------------------|--------------|---------------------|--------------|---------------------|------------|--------------------------|
| AUD-JPY          | -0.968       | $\pm 3.5e-03$       | 1.609e+02    | $\pm 3.2e+00$       | 0.99678    | 1.365e-03                |
| AUD-USD          | -0.892       | $\pm 4.3e-03$       | 2.054e+02    | $\pm 5.7e+00$       | 0.99436    | 4.187e-03                |
| CHF-JPY          | -1.006       | $\pm 3.3e-03$       | 7.612e+01    | $\pm 1.2e+00$       | 0.99731    | 1.683e-03                |
| EUR-AUD          | -1.071       | $\pm 3.2e-03$       | 5.888e+01    | $\pm 8.3e-01$       | 0.99775    | 3.374e-04                |
| EUR-CHF          | -1.124       | $\pm 6.7e-03$       | 7.751e+00    | $\pm 1.4e-01$       | 0.99117    | 3.547e-03                |
| EUR-GBP          | -1.084       | $\pm 3.6e-03$       | 2.503e+01    | $\pm 3.3e-01$       | 0.99729    | 9.155e-04                |
| EUR-JPY          | -0.954       | $\pm 1.7e-03$       | 1.010e+02    | $\pm 9.3e-01$       | 0.99921    | 3.324e-04                |
| EUR-USD          | -0.917       | $\pm 4.2e-03$       | 1.061e+02    | $\pm 2.6e+00$       | 0.99474    | 4.471e-03                |
| GBP-CHF          | -1.098       | $\pm 3.4e-03$       | 3.117e+01    | $\pm 4.0e-01$       | 0.99766    | 1.305e-03                |
| GBP-JPY          | -0.978       | $\pm 2.4e-03$       | 9.847e+01    | $\pm 1.2e+00$       | 0.99852    | 5.018e-04                |
| GBP-USD          | -0.906       | $\pm 3.3e-03$       | 9.983e+01    | $\pm 1.9e+00$       | 0.99667    | 2.486e-03                |
| GRW              | -0.866       | $\pm 6.6e-03$       | 2.803e+02    | $\pm 1.3e+01$       | 0.98589    | 1.256e-02                |
| USD-CHF          | -0.922       | $\pm 4.5e-03$       | 1.277e+02    | $\pm 3.3e+00$       | 0.99413    | 5.365e-03                |
| USD-JPY          | -0.945       | $\pm 4.7e-03$       | 9.785e+01    | $\pm 2.5e+00$       | 0.99380    | 5.314e-03                |
| Currency average | -0.99        | (8.0e-02)           | 9.20e+01     | (5.5e+01)           |            |                          |